4.5 Integration methods and volumes of revolution

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

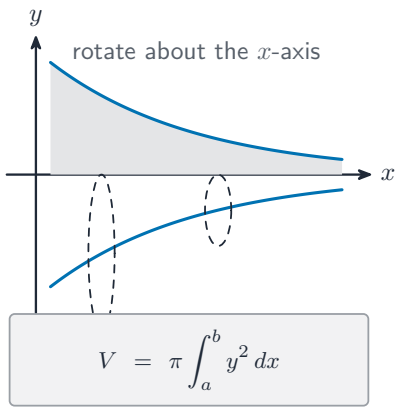
KEY WORDS substitution • integration by parts • volume of revolution

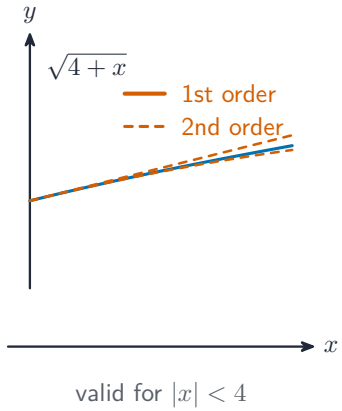
Objective

Integrate by substitution (given), by parts (including repeats and $\ln x$), and via partial fractions; and compute volumes of revolution $V = \pi \int y^2 dx$ including parametric curves. Original teaching questions for specification items P4.6.1 to P4.6.3 and P4.6.5; the exam-style part adapts reviewed WMA14 June 2025 demand.

1 Key knowledge and worked examples

- **Substitution** (usually given): convert everything to u (including dx via $\frac{du}{dx}$), integrate, substitute back.
- **Parts** : $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$. Know $\int \ln x\, dx = x\ln x - x + c$; repeated parts for $\int x^2 e^x dx$.
- **Partial fractions** integrate to logs and powers.
- **Volume of revolution** : $V = \pi \int_a^b y^2\, dx$; parametric: $V = \pi \int_{t_1}^{t_2} y^2 \frac{dx}{dt}\, dt$.





Square first; identities handle squared trig.

Worked example – parts that repeats. Find $\int e^x \cos x\,dx$.

$$\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx = e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$

$$2\int e^x \cos x\,dx = e^x(\sin x + \cos x) \implies \int e^x \cos x\,dx = \tfrac{1}{2}e^x(\sin x + \cos x) + c$$

- Check by differentiating: $\frac{1}{2}e^x(\sin x + \cos x + \cos x - \sin x) = e^x \cos x$ ✓.

Worked example – volume with an identity. $y = 3\cos x$ on $[0, \frac{\pi}{2}]$ rotates about the x-axis:

$$V = \pi \int_0^{\pi/2} 9\cos^2 x\,dx = \frac{9\pi}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi/2} = \frac{9\pi^2}{4}$$

- Check: the mean of $\cos^2$ over a half-turn is $\frac{1}{2}$, so $V \approx 9\pi \times \frac{1}{2} \times \frac{\pi}{2}$ ✓ exactly.

2 Practice

2.1 Use the substitution $u = x^2 + 1$ to find $\int x(x^2+1)^3\,dx$. [3]

2.2 Find $\int xe^{2x}\,dx$ by parts. [3]

2.3 Find $\int \ln 3x\,dx$. [3]

2.4 Find $\int \frac{4x+5}{(x+1)(x+2)}\,dx$ using partial fractions. [5]

2.5 Find the volume when $y = \sqrt{x}$ on $[0, 4]$ rotates about the x-axis. [4]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q7.)* Use the substitution $u = \tan x$ to find $\int \tan x \sec^2 x \, dx$. [4]

3.2 *(Structure adapted from Edexcel IAL WMA14 June 2025 Q9.)* The region under $y = 2\sin x$ for $0 \leq x \leq \pi$ rotates about the x-axis. Find the exact volume, showing your use of a double-angle identity. [6]