

3.6 Integration and numerical methods

Name: _____ Class: _____ Date: _____

Total: 26 marks

KEY WORDS recognition forms • sign change • iteration

Objective

Integrate e^{kx} , $\frac{1}{x}$ and $\sin / \cos kx$; recognise $\int \frac{f'(x)}{f(x)} dx$ and $\int f'(x)(f(x))^n dx$; use identities on squared trig functions; locate roots by sign change; and run iterations $x_{n+1} = f(x_n)$. Original teaching questions for specification items P3.5.1, P3.5.2, P3.6.1 and P3.6.2; the exam-style part adapts reviewed WMA13 June 2025 demand.

1 Key knowledge and worked examples

$$\int e^{kx} dx = \frac{1}{k} e^{kx} + c \quad \int \frac{1}{x} dx = \ln |x| + c$$

$$\int \cos kx dx = \frac{1}{k} \sin kx + c \quad \int \sin kx dx = -\frac{1}{k} \cos kx + c$$

Recognition forms :

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c \quad \int f'(x)(f(x))^n dx = \frac{(f(x))^{n+1}}{n+1} + c \quad (n \neq -1)$$

For squares: $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$, $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$, $\tan^2 A = \sec^2 A - 1$.

Numerical methods : if f is continuous and $f(a)f(b) < 0$, a root lies in (a, b) ; an iteration $x_{n+1} = f(x_n)$ from a start value approaches a fixed point when it converges – confirm the final accuracy with a sign change.

Worked example – identity first (adapted from Edexcel IAL WMA13 June 2025 Q5). Find $\int \sin^2 3x dx$.

- $\cos 6x \equiv 1 - 2 \sin^2 3x$, so $\sin^2 3x = \frac{1}{2}(1 - \cos 6x)$:

$$\int \sin^2 3x dx = \frac{1}{2} \int (1 - \cos 6x) dx = \frac{x}{2} - \frac{\sin 6x}{12} + c$$

- Check: differentiating gives $\frac{1}{2} - \frac{1}{2} \cos 6x = \sin^2 3x$ ✓.

Worked example – recognition. Find $\int \frac{4x}{x^2 + 3} dx$ and $\int 2x\sqrt{x^2 + 3} dx$.

- First: numerator $4x$ is the derivative of $x^2 + 3$:

$$\int \frac{4x}{x^2 + 3} dx = \ln|x^2 + 3| + c$$

- Second: with $u = x^2 + 3$, $du = 2x dx$:

$$\int 2x u^{1/2} dx = \int u^{1/2} du = \frac{2}{3} u^{3/2} + c = \frac{2}{3} (x^2 + 3)^{3/2} + c$$

- Check the second: $\frac{d}{dx} \frac{2}{3} (x^2 + 3)^{3/2} = \frac{2}{3} \cdot \frac{3}{2} (x^2 + 3)^{1/2} \cdot 2x = 2x\sqrt{x^2 + 3}$ ✓.

2 Practice

2.1 Find $\int e^{5x} dx$. [2]

2.2 Find $\int \frac{3}{x} dx$ ($x > 0$). [2]

2.3 Find $\int \cos 4x dx$. [2]

2.4 Find $\int \frac{2x + 2}{x^2 + 2x + 5} dx$. [3]

2.5 Find $\int 3x^2(x^3 + 1)^4 dx$. [3]

2.6 Show that $x^3 - 3x + 1 = 0$ has a root between 0 and 1. [2]

3 Exam-style questions

3.1 (*Adapted from Edexcel IAL WMA13 June 2025 Q5 and Q7.*)

(a) Find $\int \cos^2 2x dx$.

(b) Find $\int \frac{x}{x^2 + 4} dx$. [5]

3.2 (*Adapted from Edexcel IAL WMA13 June 2025 Q7.*) The equation $x = e^{-x}$ has exactly one positive root.

(a) Show the root lies between 0.5 and 0.6.

(b) Use the iteration $x_{n+1} = e^{-x_n}$ from $x_1 = 0.5$ to find x_3 to 3 decimal places.

(c) Verify the root is 0.567 to 3 decimal places. [7]
