1.4 Triangle trigonometry and radians

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS sine rule • cosine rule • radian • sector • ambiguous case

Objective

Use the sine rule (including the ambiguous case), the cosine rule and the area formula $\frac{1}{2}ab\sin C$, and work in radians with $s = r\theta$ and sector area $\frac{1}{2}r^2\theta$. Original teaching questions for specification items P1.3.1 and P1.3.2; the exam-style part adapts reviewed WMA11 June 2025 demand.

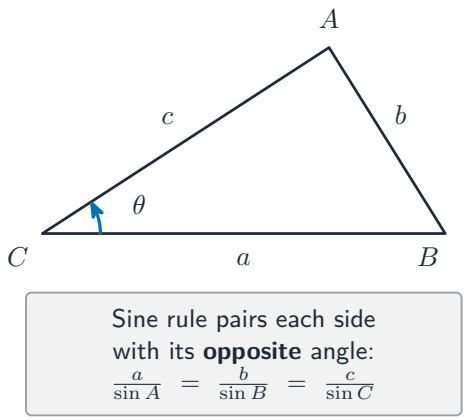
1 Key knowledge and worked examples

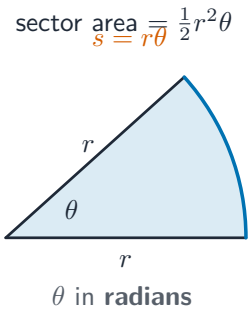
In triangle ABC with sides a, b, c opposite angles A, B, C:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc\cos A \qquad \text{area} = \tfrac{1}{2}ab\sin C$$

Quote the formula first: the method mark follows the quoted formula. The sine rule has an **ambiguous case**: $\sin\theta = \sin(180° - \theta)$, so both the acute and the obtuse angle can fit the same sine value – check the diagram.

Radians: $180° = \pi$ rad. With θ in radians, **arc length** $s = r\theta$ and **sector area** $= \frac{1}{2}r^2\theta$.





Each side pairs with its opposite angle.

Worked example – cosine rule. In triangle ABC, $b = 5$, $c = 7$ and $A = 60°$. Find a.

- Known: two sides and the included angle, so the cosine rule finds the third side:

$$a^2 = b^2 + c^2 - 2bc\cos A = 25 + 49 - 2(5)(7)\cos 60° = 74 - 70 \times \tfrac{1}{2} = 74 - 35 = 39$$

$$a = \sqrt{39} \approx 6.24$$

- Check: the answer sits between $|7 - 5| = 2$ and $7 + 5 = 12$ ✓.

Worked example – ambiguous sine rule. In triangle ABC, $A = 35°$, $a = 6$ and $b = 8$. Find the two possible values of B.

$$\frac{\sin B}{8} = \frac{\sin 35°}{6} \implies \sin B = \frac{8\sin 35°}{6} = 0.7648$$

$$B = 49.9° \quad \text{or} \quad B = 180° - 49.9° = 130.1°$$

- Both are possible because $130.1° + 35° = 165.1° < 180°$ leaves room for angle C.
- Check: with $B = 130.1°$, $C = 14.9°$, and $\sin B = \sin 49.9°$ ✓ – one sine, two angles.

2 Practice

2.1 Convert 135° to radians, giving an exact answer in terms of π. [2]

2.2 A circle has radius 10 cm. Find the arc length for a sector angle of 0.5 radians. [2]

2.3 Find the area of a sector with radius 4 cm and angle $\frac{\pi}{6}$ radians, giving an exact answer. [2]

2.4 Two sides of a triangle are 5 cm and 8 cm with included angle 32°. Find the area. [2]

2.5 In triangle ABC, $A = 80°$, $a = 12$ and $C = 50°$. Find c. [3]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA11 June 2025 Q6.)* A sector of a circle has radius 9 m and angle 1.2 radians.

(a) Find the area of the sector.

(b) Find the perimeter of the sector. [4]

3.2 A triangular garden plot has sides of 11 m and 15 m with included angle 0.9 radians.

(a) Find the area of the plot.

(b) Find the length of the third side, giving your answer to 3 significant figures. [5]