1.2.1 Simultaneous equations and inequalities

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS simultaneous equations • substitution • inequality • dotted boundary

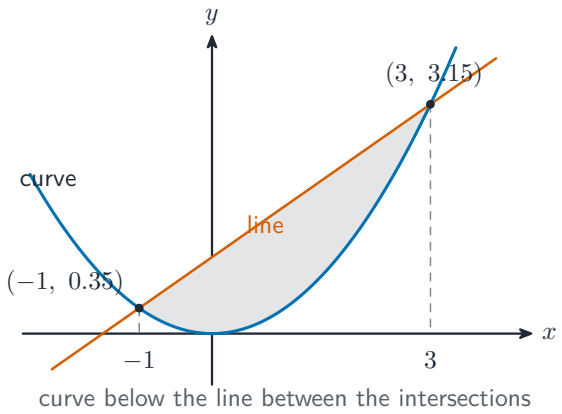
Objective

Solve simultaneous equations by substitution (including a line meeting a curve) and solve linear and quadratic inequalities, interpreting them graphically. Original teaching questions for specification items P1.1.6 to P1.1.9; the exam-style part adapts reviewed WMA11 June 2025 demand.

1 Key knowledge and worked examples

Simultaneous equations by **substitution** : rearrange the linear equation for one variable, substitute into the other equation, solve, then substitute back. A line meeting a curve gives a quadratic in one unknown; its roots are the intersection x-coordinates – the geometrical meaning of the algebra.

Inequalities keep their sign under add/subtract and under division by a positive number; dividing by a negative number reverses the sign. For a quadratic inequality, sketch the parabola and read off where it sits above or below the axis. When **sketching** a region, use a **dotted boundary** for strict inequalities and a **solid boundary** for $\leq$ or $\geq$.



Between the intersections the curve sits below the line.

Worked example – line meets curve. Find the intersections of $y = x + 1$ and $y = x^2 - 1$.

- Substitute the line's y into the curve:

$$x + 1 = x^2 - 1 \implies x^2 - x - 2 = 0 \implies (x-2)(x+1) = 0 \implies x = 2 \text{ or } x = -1$$

- Substitute back into $y = x + 1$: the points are $(2, 3)$ and $(-1, 0)$.
- Check in the curve: $2^2 - 1 = 3$ ✓ and $(-1)^2 - 1 = 0$ ✓.

Worked example – quadratic inequality. Solve $x^2 + 4x - 5 \geq 0$.

- Factorise: $(x + 5)(x - 1) \geq 0$. The parabola opens upward with roots -5 and 1.
- Upward parabolas sit on or above the axis **outside** their roots:

$$x \leq -5 \quad \text{or} \quad x \geq 1$$

- Check $x = 0$ (excluded): $-5 \geq 0$ false ✓; $x = 2$ (included): $4 + 8 - 5 = 7 \geq 0$ ✓.

2 Practice

2.1 Show that the line $y = 2x - 3$ does not meet the curve $y = x^2$. [3]

2.2 Solve $3x - 2 > x + 8$. [2]

2.3 Solve $x^2 \leq 9$. [2]

2.4 Solve $x^2 > 4x + 5$. [3]

2.5 Find the range of x for which the curve $y = x^2$ is below the line $y = 2x + 3$. [3]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA11 June 2025 Q7(c).)* The curves C_1: $y = x^2 - 4x + 5$ and C_2: $y = 2x - 1$ intersect at two points. Use algebra to find the exact coordinates of both. [6]

3.2 Solve the inequality $\dfrac{2}{x} < x$, where $x \neq 0$. [5]