9.6 Mixed D1 practice

Name: _____ Class: _____ Date: _____ **Total: 29 marks**

KEY WORDS algorithm • graph

Mixed D1 practice

Objective

Combine the unit's algorithms on fuller problems: sorting with packing, networks with routes, and formulation with graphical solution (objectives 1.1–5.3).

1 Key knowledge and worked examples

Combinations that recur. Sort-then-pack (first-fit decreasing needs the sorted list). Complete a shortest-distance table by inspection/Dijkstra before nearest neighbour. Gantt charts after the forward/backward passes. Integer vertex checks after the graphical optimum.

Worked example (original). Items 6, 4, 7, 3, 5 in bins of capacity 10: sorted descending 7, 6, 5, 4, 3, then first-fit: 7→b1 (7), 6→b1 fails ($13 > 10$) → b2 (6), 5→b1 fails (12), b2 fails (11) → b3 (5), 4→b1 fails (11) → b2 (10), 3→b1 (10): bins (7,3), (6,4), (5) – 3 bins. The total weight 25 with capacity 10 forces at least 3 bins, so this packing is optimal.

2 Practice

2.1 Quick sort 5, 9, 2, 6, stating each pivot and the list after each partition. [3]

2.2 A network has edges RS 4, ST 1, RT 3, SU 5, TU 2. Find the MST by Prim from R. [3]

2.3 For the constraints $2x + y \leq 10$, $x + y \leq 8$, $x, y \geq 0$: maximise $x + 2y$ by the vertex method. [4]

3 Exam-style questions

3.1 *(Original.)* A rescue team must inspect every road between villages P, Q, R, S, T starting and ending at P. The roads and their lengths (km) are PQ 4, QR 2, RS 3, ST 5, PR 6, PS 2, QT 4, RT 1. (a) Find the total of all the road lengths. (b) Identify the odd-degree villages. (c) Find the least-weight pairing of the odd villages, listing every pairing. (d) Find the length of the shortest inspection route. [10]

3.2 *(Original.)* A project has activities A (2), B (5), C (3 after A), D (4 after B), E (2 after C and D), F (1 after A). (a) Find the project duration and the critical path. (b) Find the total float of F. (c) The network is redrawn as a linear program limiting "workers on day 1" – the LP is maximise $W = x + y$ with $x \leq 3$, $y \leq 2$, $x + y \leq 4$: solve it and state the integer optimum. [9]

Solutions

2.1 Pivot (position 2 of 4) = 9: less = 5, 2, 6; greater = none → 5, 2, 6, **9**. Sublist 5, 2, 6 (pivot 2): less none; greater = 5, 6 → **2**, 5, 6. Sublist 5, 6 (pivot 5): less none; greater = 6 → **5**, 6. Final: 2, 5, 6, 9.

2.2 From R: edges RS 4, RT 3 – take RT(3). From {R,T}: edges to new vertices are RS 4, TS 1, TU 2 – take TS(1). From {R,T,S}: edges to U are SU 5 and TU 2 – take TU(2). MST = RT + TS + TU, total 6.

2.3 Vertices: $(0,0)$: 0; $(5,0)$ $(2x+y=10)$: 5; $(2,6)$ (intersection: $2x+y=10, x+y=8$ $\to x=2, y=6$): $2+12=14$; $(0,8)$: 16. Maximum 16 at $(0,8)$.

3.1 (a) Total = 4 + 2 + 3 + 5 + 6 + 2 + 4 + 1 = 27. (b) Degrees: P 3 (PQ, PR, PS), Q 3 (PQ, QR, QT), R 4 (QR, RS, PR, RT), S 3 (RS, ST, PS), T 3 (ST, QT, RT): odd villages P, Q, S, T. (c) Pairings of {P, Q, S, T} (weights are shortest paths where no direct edge exists): PQ + ST = 4 + 5 = 9; PS + QT = 2 + 4 = 6; PT + QS = 7 + 5 = 12 (PT via PS–ST or PR–RT, both 7; QS via QR–RS, 5). Least = PS + QT = 6. (d) Route length = 27 + 6 = 33.

3.2 (a) Forward pass: A 0–2, F 2–3 (F after A: starts 2, ends 3), B 0–5, C starts 2 ends 5, D starts 5 ends 9, E starts 9 ends 11. Duration 11; critical path B–D–E (5 + 4 + 2 = 11). (b) F's latest finish = latest start of nothing downstream (F is an end activity): latest finish 11, earliest start 2, duration 1: float = 11 − 2 − 1 = 8 days. (c) Vertices of $\{x \le 3, y \le 2, x+y \le 4\}$: $(0,0)$, $(3,0)$, $(2,2)$, $(0,2)$: $W = 0, 3, 4, 2$: optimum $W = 4$ at $(2,2)$ – already integral ✓.

Local review guidance: pairing weights use shortest paths when the direct edge is absent; end activities' latest finish is the project duration; integer optima confirmed at neighbouring lattice points. This is authored classroom guidance.