# 9.4 Critical path analysis

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

**KEY WORDS** critical path • total float

## Critical path analysis

## Objective

Build activity-on-arc networks from precedence tables (with dummies), run forward and backward passes for event times, find the critical path, compute floats and draw Gantt charts (objectives 4.1–4.4).

## 1 Key knowledge and worked examples

**Pipeline.** Precedence table → activity network (activities on arcs, events as nodes; dummies where dependencies need them). Forward pass: earliest time of an event = max over arriving activities of (earliest of tail + duration). Backward pass: latest time = min over leaving activities of (latest of head − duration). Critical activities have zero total float: latest finish − earliest start − duration = 0; the critical path runs start to finish through them and its length is the project duration. Gantt bars run from each activity's earliest start for its duration, with float as trailing slack.

**Worked example (original).** A (3), B (4), C (4 after A), D (1 after B), E (5 after C and D). Chain A–C–E = 12, chain B–D–E = 10: critical path A–C–E, project duration 12 days; D's total float = 12 − (4 + 1 + 5) = 2 days.

## 2 Practice

**2.1** Activities A (2), B (5), C (3 after A), D (1 after A and B). Draw the precedence table as an activity network description (list arcs into and out of each event) and find the project duration. [4]

**2.2** For a project with earliest event times 0, 4, 7, 9 and latest times 0, 4, 7, 9 along one chain, state the float of each activity on the chain. [2]

**2.3** An activity lasts 6 days, its earliest start is day 5 and its latest finish is day 16. Find its total float. [2]

## 3 Exam-style questions

**3.1** *(Original.)* A project has activities A (4), B (3), C (6 after A), D (2 after A and B), E (5 after C and D), F (3 after D). (a) Draw an activity network for the project (from the precedence table), using a dummy where necessary, and describe where the dummy is needed. (b) Carry out a forward pass and a backward pass, listing the earliest and latest times for each event. (c) Find the critical path and the duration of the project. (d) Find the total float of F. [11]

**3.2** *(Original.)* Using the project of 3.1: (a) draw a Gantt chart for the project, labelling the critical activities; (b) state the minimum number of workers needed to avoid delaying any critical activity; (c) explain why F can start as late as day 12. [6]

## Solutions

**2.1** Events: start s; A: s→e1 (2), B: s→e2 (5); C: e1→e3 (3); D: e1→e3 and e2→e3 – D depends on both A and B, so e1 must connect to e3 and e2 to e3 (a dummy from e2 to e1 if drawn with D leaving a shared node). Durations: A–C = 2 + 3 = 5; B = 5; D needs both done: D's earliest start = max(2, 5) = 5. Project duration = max(5, 5 + 1) = 6 days.

**2.2** Earliest = latest at every event: every activity on the chain is critical, float 0.

**2.3** Float = 16 − 5 − 6 = 5 days.

**3.1** (a) Arcs: A from start to event 2; B from start to event 3; C from event 2 to event 4; D depends on A and B, so a dummy (0) runs from event 2 to event 3 and D leaves event 3 – but D must end at its OWN event 5 (time 4 + 2 = 6), since F follows D alone and must not inherit C's dependency. A dummy from event 5 to event 4 carries D into E's start (event 4 = max(C's 10, D's 6) = 10); E runs from event 4 to event 6; F runs from event 5 to event 7; a dummy from event 6 to event 7 gives the single finish. (b) Forward pass: event 1 = 0; event 2 = 4 (A); event 3 = max(3, 4 + 0) = 4; event 5 = 4 + 2 = 6 (D); event 4 = max(4 + 6, 6 + 0) = 10; event 6 = 10 + 5 = 15; event 7 = max(15 + 0, 6 + 3) = 15. Backward pass: event 7 = 15; event 6 = 15; event 4 = 10; event 5 = min(latest of event 4 − 0, latest of event 7 − 3) = min(10, 12) = 10; event 3 = 10 − 2 = 8; event 2 = min(8 − 0, 10 − 6) = 4; event 1 = min(4 − 4, 8 − 3) = 0 ✓. (c) Critical events: 1 (0/0), 2 (4/4), 4 (10/10), 6 (15/15), 7 (15/15): critical path A–C–E, duration 15 days. (d) F's earliest start is event 5's earliest time 6, its latest finish 15: float = 15 − 6 − 3 = 6 days.

**3.2** (a) Bars: A from 0 for 4; B from 0 for 3; C from 4 for 6; D from 4 for 2; E from 10 for 5; F from 6 for 3 with its slack after it. Critical activities A, C, E on one unbroken chain. (b) One worker suffices for the critical chain if nothing else overlaps it in time – but B (0–3) and D (4–6) overlap A (0–4) and C (4–10): at least 2 workers are needed. (c) F's latest finish is 15 and it lasts 3 days, so it can start as late as 15 − 3 = 12 (its earliest start is 6, giving the 6-day float).

Local review guidance: dummies carry exactly the dependency that shared endpoints cannot; forward pass takes maxima, backward pass minima; float from latest finish − earliest start − duration; Gantt bars from earliest starts with slack trailing. This is authored classroom guidance.