8.6 Normal approximations and thresholds

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS continuity correction • normal approximation

Normal approximations and thresholds

Objective

Approximate binomial (and Poisson) distributions with the Normal distribution, apply the continuity correction, solve threshold problems for r and t, and combine with testing (objectives 1.3, 3.2, 4.6).

1 Key knowledge and worked examples

Approximation. For large n: $B(n,p) \approx N(np,\ np(1-p))$ (Poisson when instead p is small). "At least t" uses $t-0.5$; "at most r" uses $r+0.5$. Threshold searches: accumulate exact binomial probabilities for small n; standardise with the correction for large n and invert $z = \frac{x-0.5-\mu}{\sigma}$.

Worked example – thresholds both ways (WST02 June 2025 Q7c–d). $X \sim B(40, 0.3)$: largest r with $P(X < r) < 0.05$ (fewer than r): $P(X \leq 6) = 0.0238 < 0.05$ but $P(X \leq 7) = 0.0553 > 0.05$, so $r = 7$. For 200 customers: $B(200, 0.3) \approx N(60, 42)$; $P(X \geq t) = 0.9474$ means $\frac{t-0.5-60}{\sqrt{42}} = -1.62$, so $t = 0.5 + 60 - 1.62\sqrt{42} = 50.001$: $t = 50$.

2 Practice

2.1 $X \sim B(120, 0.4)$. State the approximating Normal distribution. [2]

2.2 Using that approximation, find $P(X > 55)$, with continuity correction. [3]

2.3 A Poisson variable has mean 100. State its Normal approximation and estimate $P(X < 90)$. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WST02 June 2025 Q7, condensed.)* 30% of a random sample of 40 customers buying glasses also buy insurance; X is the number buying insurance. (a) State the distribution of X and one necessary assumption. (b) Find the largest r for which $P(X < r) < 0.05$ (fewer than r). (c) For a second sample of 200 customers, using a suitable Normal approximation, find the value of t such that $P(\text{at least } t \text{ of the 200 buy insurance}) = 0.9474$ (4 dp), showing all working. [9]

3.2 *(Original.)* A telephone line receives calls at an average rate of 4 per minute. Using a Normal approximation, estimate the probability that fewer than 200 calls arrive in one hour. [5]

Solutions

2.1 $N(48,\ 28.8)$ (mean 120×0.4; variance $120 \times 0.4 \times 0.6 = 28.8$).

2.2 $P(X > 55) = P(Z > \frac{55.5-48}{\sqrt{28.8}}) = P(Z > 1.398) = 1 - 0.9192 = 0.081$.

2.3 $N(100, 100)$; $P(X < 90) = P(Z < \frac{89.5-100}{10}) = P(Z < -1.05) = 0.1469$ (0.147).

3.1 (a) $X \sim B(40, 0.3)$; customers buy insurance independently with a constant probability 0.3. (b) $P(X < r) = P(X \leq r - 1)$: $P(X \leq 6) = 0.0238 < 0.05$ but $P(X \leq 7) = 0.0553 > 0.05$, so the largest such r is 7 (P(fewer than 7) is the last one under 0.05) (c) $B(200, 0.3) \approx N(60,\ 42)$ with $\sigma = \sqrt{42}$. "At least t" $\rightarrow$ boundary $t - 0.5$: $P\left(Z > \frac{t-0.5-60}{\sqrt{42}}\right) = 0.9474$ means $\frac{t-0.5-60}{\sqrt{42}} = -1.62$, so $t = 0.5+60-1.62(6.481) = 50.001$: $t = 50$.

3.2 Calls per hour: $\text{Po}(240) \approx N(240, 240)$ with $\sigma = \sqrt{240} = 15.49$. Fewer than 200 $\rightarrow$ boundary 199.5: $P\left(Z < \frac{199.5-240}{15.49}\right) = P(Z < -2.615)$. From tables, $\Phi(-2.62) = 0.0044$: the estimate is awrt 0.0044.

Local review guidance: the correct ± 0.5 boundary for the phrasing ("at least", "fewer than"); z-values to 2 dp and inverted cleanly to the threshold; r-searches closed by the two straddling probabilities, both quoted. This is authored classroom guidance.