6.6 The Normal distribution

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS Normal distribution

The Normal distribution

Objective

Standardise Normal variables in both directions, use symmetry, handle binomial-style "exactly r of n" follow-ups, and finish two-condition problems with simultaneous equations (Normal objective).

1 Key knowledge and worked examples

Standardising. $X \sim N(\mu, \sigma^2)$: $Z = \frac{X-\mu}{\sigma}$ is standard Normal. Forward: probability from the table. Backward: z from the table, then $x = \mu + z\sigma$. Symmetry: $P(X < \mu - a) = P(X > \mu + a)$. Exactly r of n independent items: $\binom{n}{r}p^r(1-p)^{n-r}$. Two conditions on one unknown: equate the two z-expressions.

Worked example – bags of flour (WST01 June 2025 Q6). Small bags $X \sim N(502, 3^2)$: $P(X > 508) = P(Z > 2) = 0.0228$; $P(496 < X < 508) = 1 - 2(0.0228) = 0.9544$; of 4 bags exactly 2 over 508 g: $6(0.0228)^2(0.9772)^2 = 0.00298$. Large bags $Y \sim N(1024, \sigma^2)$ with 85th percentile 1038.51: $\sigma = \frac{14.51}{1.0364} = 14$. And $P(X > k) = P(Y < 2k)$ gives $\frac{k-502}{3} = \frac{1024-2k}{14}$, so $k = 505$ and $p = P(Z > 1) = 0.159$.

2 Practice

2.1 $Z \sim N(0, 1)$. Write down $P(Z > 1.5)$ and hence $P(Z < -1.5)$. [2]

2.2 $X \sim N(40, 6^2)$. Find $P(X > 46)$. [3]

2.3 $X \sim N(50, \sigma^2)$ and $P(X > 56) = 0.1587$. Find σ. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WST01 June 2025 Q6, condensed.)* The weight X grams in a small bag of flour is $N(502, 3^2)$. (a) Find $P(X > 508)$ and hence $P(496 < X < 508)$. (b) A random sample of 4 small bags is taken; find the probability that exactly 2 of them contain more than 508 g. (c) The weight Y grams in a large bag is $N(1024, \sigma^2)$ and the 85th percentile of Y is 1038.51; show that $\sigma = 14$ to the nearest gram. [9]

3.2 *(Edexcel IAL WST01 June 2025 Q6d.)* Given that $P(X > k) = P(Y < 2k)$, where $X \sim N(502, 3^2)$ and $Y \sim N(1024, 14^2)$, find k and the common probability. [5]

Solutions

2.1 $P(Z > 1.5) = 0.0668$; by symmetry $P(Z < -1.5) = 0.0668$.

2.2 $z = \frac{46-40}{6} = 1$: $P(X > 46) = P(Z > 1) = 0.1587$.

2.3 $P\left(Z > \frac{56-50}{\sigma}\right) = 0.1587 \implies z = 1 \implies \sigma = 6$.

3.1 (a) $P(X > 508) = P\left(Z > \frac{508-502}{3}\right) = P(Z > 2) = 0.0228$. Since 496 and 508 sit symmetrically two SDs either side of 502, $P(496 < X < 508) = 1 - 2(0.0228) = 0.9544$. (b) With $p = 0.0228$: $\binom{4}{2}p^2(1-p)^2 = 6(0.0228)^2(0.9772)^2 = 0.00298$. (c) The 85th percentile has $z = 1.0364$: $\frac{1038.51-1024}{\sigma} = 1.0364$ gives $\sigma = \frac{14.51}{1.0364} = 14.0$, i.e. $\sigma = 14$ ✓.

3.2 Standardise both: $P(X > k) = P\left(Z > \frac{k-502}{3}\right)$ and $P(Y < 2k) = P\left(Z < \frac{1024-2k}{14}\right)$. For these two probabilities to be equal, the two z-values must be EQUAL – a right tail and a left tail carry the same probability exactly when they are the same distance from 0 in the same direction: $\frac{k-502}{3} = \frac{1024-2k}{14}$. Clearing: $14k - 7028 = 3072 - 6k$, so $20k = 10100$ and $k = 505$. Then $p = P\left(Z > \frac{505-502}{3}\right) = P(Z > 1) = 0.159$.

Local review guidance: standardisations with the correct sign pairings (equal probabilities $\implies$ equal z for a $>$ and a $<$ on opposite sides); exact table values with 4 dp tolerance; binomial term with the correct coefficient. This is authored classroom guidance.