6.5 Discrete random variables

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS discrete random variable • expectation

Discrete random variables

Objective

Use probability functions and cumulative functions of discrete random variables, compute E(X), E(X^2), Var(X), apply linear coding, and quote the discrete uniform results (objectives 5.1–5.4).

1 Key knowledge and worked examples

Routine. Probabilities sum to 1 (find missing values). $F(x_0) = P(X \le x_0)$. Solve inequalities for X BEFORE summing probabilities. $E(X) = \sum xp(x)$, $E(X^2) = \sum x^2 p(x)$, $\mathrm{Var}(X) = E(X^2) - [E(X)]^2$; $E(aX+b) = aE(X) + b$, $\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$. Uniform on $1..n$: $E(X) = \frac{n+1}{2}$, $\mathrm{Var}(X) = \frac{n^2-1}{12}$.

Worked example – the full routine (WST01 June 2025 Q1). X on 1–4 with probabilities 0.3, 0.2, 0.35, 0.15: $F(3.5) = 0.85$; $3X - 3 < X + 2 \Rightarrow X < 2.5 \Rightarrow P = 0.5$; $E(X) = 2.35$; $E(X^2) = 6.65$; $\mathrm{Var}(X) = 1.1275$; $\mathrm{Var}(5-2X) = 4(1.1275) = 4.51$.

2 Practice

2.1 X has $P(X = x) = \frac{kx}{6}$ for $x = 1, 2, 3$. Find k, $E(X)$ and $\mathrm{Var}(X)$. [4]

2.2 A fair die is rolled once. Using the uniform results, write down $E(X)$ and $\mathrm{Var}(X)$. [2]

2.3 Y has $E(Y) = 4$ and $\mathrm{Var}(Y) = 2.5$. Find $E(3Y-2)$ and $\mathrm{Var}(3Y-2)$. [2]

3 Exam-style questions

3.1 *(Edexcel IAL WST01 June 2025 Q1.)* The discrete random variable X has probability distribution

x	**1**	**2**	**3**	**4**
$P(X = x)$	0.3	0.2	0.35	0.15

(a) Find $F(3.5)$. (b) Find $P(3X - 3 < X + 2)$. (c) Find $E(X)$. (d) Show that $\mathrm{Var}(X) = 1.1275$. (e) Find $\mathrm{Var}(5 - 2X)$. [9]

3.2 *(Original.)* A game scores X where $P(X = 0) = 0.25$, $P(X = 1) = 0.5$ and $P(X = 4) = 0.25$. The winnings are $W = 10X - 3$. (a) Find $E(W)$ and $\mathrm{Var}(W)$. (b) Find $P(W > E(W))$. [6]

Solutions

2.1 $\frac{k}{6}(1+2+3)=1$ gives $k=1$: so the probabilities are $\frac{1}{6},\frac{2}{6},\frac{3}{6}$. $E(X)=\frac{1+4+9}{6}=\frac{14}{6}$; $E(X^2)=\frac{1+8+27}{6}=\frac{36}{6}=6$; $\mathrm{Var}(X)=6-\left(\frac{14}{6}\right)^2=6-\frac{196}{36}=\frac{5}{9}$.

2.2 Uniform on 1–6: $E(X)=\frac{7}{2}=3.5$; $\mathrm{Var}(X)=\frac{36-1}{12}=\frac{35}{12}\approx 2.92$.

2.3 $E(3Y-2)=3(4)-2=10$; $\mathrm{Var}(3Y-2)=3^2(2.5)=22.5$.

3.1 (a) $F(3.5)=P(X\le 3.5)=0.3+0.2+0.35=0.85$. (b) $3X-3<X+2$ simplifies to $2X<5$, i.e. $X<2.5$, so $P=P(1)+P(2)=0.5$. (c) $E(X)=0.3+0.4+1.05+0.6=2.35$. (d) $E(X^2)=0.3+0.8+3.15+2.4=6.65$; $\mathrm{Var}(X)=6.65-2.35^2=6.65-5.5225=1.1275$ ✓. (e) $\mathrm{Var}(5-2X)=(-2)^2(1.1275)=4.51$.

3.2 (a) $E(X)=0+0.5+1=1.5$; $E(W)=10(1.5)-3=12$. $E(X^2)=0+0.5+4=4.5$; $\mathrm{Var}(X)=4.5-2.25=2.25$; $\mathrm{Var}(W)=100(2.25)=225$. (b) $W>12\Leftrightarrow 10X-3>12\Leftrightarrow X>1.5\Leftrightarrow X=4$: $P=0.25$.

Local review guidance: probabilities totalled to 1 before anything else; inequality solved for X first; variance via $E(X^2)-E(X)^2$ and coding scalars squared. This is authored classroom guidance.