3.2 Modulus and combined transformations

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

KEY WORDS modulus • combined transformations

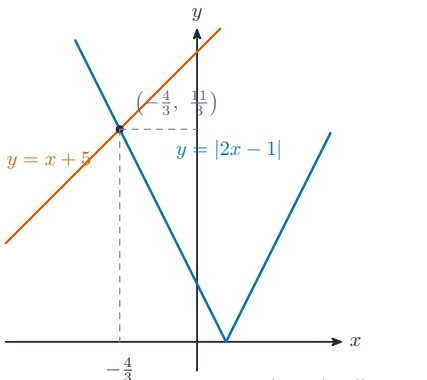
Objective

Sketch and solve equations with the modulus function, sketch $y = |f(x)|$ and $y = f(|x|)$ from a given graph, and combine two transformations on one graph. Original teaching questions for specification items P3.1.3 and P3.1.4.

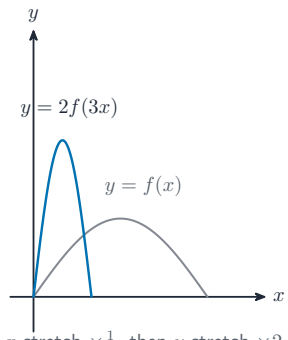
1 Key knowledge and worked examples

The **modulus** $|x|$ equals x when $x \geq 0$ and $-x$ when $x < 0$, so $|ax + b|$ never goes below the x-axis: its graph is the line $y = ax + b$ with the below-axis part reflected up. Solve modulus equations **by cases**, checking every candidate against its case's condition.

From $y = f(x)$ you should also sketch $y = |f(x)|$ (reflect below-axis parts up) and $y = f(|x|)$ (keep the $x \geq 0$ half and mirror it to the left). **Combined transformations** stack the P1 rules: e.g. $y = 2f(3x)$ first stretches parallel to the x-axis by $\frac{1}{3}$, then parallel to the y-axis by 2.



second intersection at $(6,\ 11)$, off-screen



x-stretch $\times\frac{1}{3}$, then y-stretch $\times 2$

The V vertex sits at the root of the inside expression.

Worked example – modulus equation (the specification's own example). Solve $|2x - 1| = x + 5$.

- Case $x \geq \frac{1}{2}$: $2x - 1 = x + 5 \implies x = 6$ (valid: $6 \geq \frac{1}{2}$ ✓).
- Case $x < \frac{1}{2}$: $1 - 2x = x + 5 \implies x = -\frac{4}{3}$ (valid: $-\frac{4}{3} < \frac{1}{2}$ ✓).
- Solutions: $x = 6$ and $x = -\frac{4}{3}$. Check: $|11| = 11$ ✓ and $|-\frac{11}{3}| = \frac{11}{3} = -\frac{4}{3} + 5$ ✓.

2 Practice

2.1 Sketch $y = |x - 2|$, marking the vertex. [2]

2.2 Solve $|x - 3| = 5$. [2]

2.3 Solve $|2x + 1| = 7$. [2]

2.4 Describe the transformations mapping $y = f(x)$ onto $y = 3f(x) - 2$. [2]

2.5 Given $y = f(x)$ passes through $(2, 5)$, state the image of this point on $y = f(2x) + 3$. [3]

3 Exam-style questions

3.1 Solve $|x^2 - 4| = 3x + 2$, giving exact answers and checking each one. [5]

3.2 The graph of $y = f(x)$ has a maximum at $(1, 4)$ and crosses the x-axis at $x = -1$ and $x = 3$. Sketch, on separate axes, $y = |f(x)|$ and $y = f(|x|)$, labelling key points. [5]

Solutions

2.1 A V with vertex at $(2, 0)$, arms of gradient ± 1.

2.2 $x - 3 = \pm 5$: $x = 8$ or $x = -2$.

2.3 $2x + 1 = \pm 7$: $x = 3$ or $x = -4$.

2.4 Stretch parallel to the y-axis, factor 3, then translation $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ (down 2).

2.5 $y = f(2x)$ halves x-coordinates: $(2, 5) \to (1, 5)$; adding 3 moves it up: $(1, 8)$.

3.1 Case $x^2 - 4 \geq 0$ ($|x| \geq 2$): $x^2 - 4 = 3x + 2 \implies x^2 - 3x - 6 = 0 \implies x = \frac{3 \pm \sqrt{33}}{2}$. The $+$ root $= 4.37 \geq 2$ ✓; the $-$ root $= -1.37$ fails $|x| \geq 2$ ×. Case $x^2 - 4 < 0$ ($|x| < 2$): $4 - x^2 = 3x + 2 \implies x^2 + 3x - 2 = 0 \implies x = \frac{-3 \pm \sqrt{17}}{2}$. The $+$ root $= 0.56$, inside $(-2, 2)$ ✓; the $-$ root $= -3.56$ fails ×. Solutions: $x = \dfrac{3 + \sqrt{33}}{2}$ and $x = \dfrac{-3 + \sqrt{17}}{2}$. Check the second: $|0.31 - 4| = 3.69$ and $3(0.56) + 2 = 3.69$ ✓.

3.2 $y = |f(x)|$: for $-1 \leq x \leq 3$ the original sits below zero between the roots, so reflect that dip above the axis – maxima at $(1, 4)$ and the reflected dip peaking at $(1, 4)$ only if the minimum was exactly -4; otherwise label the reflected minimum as $|\text{min}|$, keeping the roots $x = -1, 3$ on the axis. $y = f(|x|)$: keep the $x \geq 0$ half (crossing at $x = 3$, falling from the maximum at $x = 1$) and mirror it into $x < 0$, so the maximum appears at both $x = \pm 1$ and the crossing at $x = \pm 3$.

Local review guidance: correct case split with its condition; exact valid roots; each root checked against its case. This is authored classroom guidance.