2.7 Trigonometric identities and equations

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS identity • trigonometric equation

Objective

Use $\tan\theta \equiv \sin\theta/\cos\theta$ and $\sin^2\theta + \cos^2\theta \equiv 1$ to reduce trigonometric equations, and solve them in a given interval in degrees or radians. Original teaching questions for specification items P2.6.1 and P2.6.2; the exam-style part adapts reviewed WMA12 June 2025 demand.

1 Key knowledge and worked examples

Two identities, used to collapse an equation into one function:

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta \equiv 1$$

Method: replace tan by sin / cos (multiply through by cos if needed), replace $\sin^2$ by $1 - \cos^2$ (or the reverse), solve the (often quadratic) equation in one function, then read every solution in the given interval from the graph's symmetry. Keep the interval's units throughout.

Worked example – hidden quadratic. Solve $2\sin^2\theta = \cos\theta + 1$ for $0 \le \theta < 360°$.

- Replace $\sin^2\theta$:

  $$2(1-\cos^2\theta) = \cos\theta+1 \implies 2\cos^2\theta+\cos\theta-1 = 0 \implies (2\cos\theta-1)(\cos\theta+1) = 0$$

- $\cos\theta = \frac{1}{2}$: $\theta = 60°, 300°$. $\cos\theta = -1$: $\theta = 180°$.
- All solutions: $60°, 180°, 300°$. Check one: $\theta = 60°$: $2(0.75) = 1.5$ and $0.5 + 1 = 1.5$ ✓.

Worked example – factoring out. Solve $\tan x = \sin x$ for $0 \le x \le 360°$.

- With $\tan x = \sin x/\cos x$:

  $$\frac{\sin x}{\cos x} = \sin x \implies \sin x = \sin x\cos x \implies \sin x(1-\cos x) = 0$$

- $\sin x = 0$: $x = 0°, 180°, 360°$; $\cos x = 1$: $x = 0°$ (already listed).

- Solutions: $0°, 180°, 360°$. Check $x = 90°$ is NOT a solution: $\tan 90°$ undefined ✓ and dividing by $\cos x$ at the start would have lost $x = 90°$ candidates – factor instead of dividing.

2 Practice

2.1 Simplify $\dfrac{\sin\theta\cos\theta}{\tan\theta}$. [2]

2.2 Solve $\sin\theta = -\frac{1}{2}$ for $0 \leq \theta < 360°$. [3]

2.3 Solve $\tan 2\theta = 1$ for $0 \leq \theta < 180°$. [4]

2.4 Given $\sin\theta = 0.6$ with θ acute, find $\cos\theta$ and $\tan\theta$. [3]

2.5 Solve $2\cos^2\theta + \cos\theta - 1 = 0$ for $0 \leq \theta < 360°$. [4]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q8(i).)* Solve, for $0 \leq x < \pi$, the equation $\tan\left(2x - \frac{\pi}{4}\right) = \sqrt{3}$, giving the answers in radians in the form $k\pi$ where k is a rational constant to be found. [4]

3.2 *(Adapted from the same question.)* Solve, for $0 \leq \theta < 360°$, the equation $4\sin\theta\tan\theta = 3\cos\theta$, giving your answers to one decimal place. [6]

Solutions

2.1 $\dfrac{\sin\theta\cos\theta}{\sin\theta/\cos\theta} = \cos^2\theta$.

2.2 Third and fourth quadrants: $\theta = 180° + 30° = 210°$ and $360° - 30° = 330°$.

2.3 2θ runs over $[0°, 360°)$: $2\theta = 45°$ or $225°$, so $\theta = 22.5°$ or $112.5°$.

2.4 $\cos\theta = \sqrt{1 - 0.36} = 0.8$ (acute, so positive); $\tan\theta = 0.6/0.8 = 0.75$.

2.5 $(2\cos\theta - 1)(\cos\theta + 1) = 0$: $\cos\theta = \frac{1}{2}$ gives $60°, 300°$; $\cos\theta = -1$ gives $180°$. Solutions: $60°, 180°, 300°$.

3.1 $\tan\left(2x - \frac{\pi}{4}\right) = \sqrt{3} = \tan\frac{\pi}{3}$:

$$2x - \frac{\pi}{4} = \frac{\pi}{3} + k\pi \implies 2x = \frac{7\pi}{12} + k\pi \implies x = \frac{7\pi}{24} + \frac{k\pi}{2}$$

In $0 \le x < \pi$: $x = \dfrac{7\pi}{24}$ $(k = 0)$ and $x = \dfrac{19\pi}{24}$ $(k = 1)$. Check: at $x = \frac{7\pi}{24}$, $2x - \frac{\pi}{4} = \frac{\pi}{3}$ and $\tan\frac{\pi}{3} = \sqrt{3}$ ✓.

3.2 Replace $\tan\theta$ and multiply through by $\cos\theta$:

$$\frac{4\sin^2\theta}{\cos\theta} = 3\cos\theta \implies 4\sin^2\theta = 3\cos^2\theta \implies 4(1 - \cos^2\theta) = 3\cos^2\theta \implies \cos^2\theta = \frac{4}{7}$$

$$\cos\theta = \pm\frac{2}{\sqrt{7}} = \pm 0.7559$$

- $\cos\theta = 0.7559$: $\theta = 40.9°$ and $360° - 40.9° = 319.1°$.
- $\cos\theta = -0.7559$: $\theta = 180° - 40.9° = 139.1°$ and $180° + 40.9° = 220.9°$. Solutions: $40.9°, 139.1°, 220.9°, 319.1°$ (1 d.p.). Check $40.9°$: $\sin = 0.654$, $\tan = 0.865$: $4(0.654)(0.865) = 2.26$ and $3(0.756) = 2.27$ ✓ (rounding).

Local review guidance: identity applied to reach one function; correct root values; all interval solutions listed with quadrant reasoning. This is authored classroom guidance.