2.5 Binomial expansion

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS binomial expansion • binomial coefficient

Objective

Expand $(a+bx)^n$ for positive integer n using binomial coefficients, match given coefficients, and use expansions for estimation. Original teaching questions for specification item P2.4.5; the exam-style part adapts reviewed WMA12 June 2025 demand.

1 Key knowledge and worked examples

The **binomial expansion** for a positive integer n:

$$(a+bx)^n = a^n + \binom{n}{1}a^{n-1}(bx) + \binom{n}{2}a^{n-2}(bx)^2 + \cdots + (bx)^n$$

with $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$ (also written ${}^nC_r$). Powers of a fall, powers of bx rise, and the coefficients are row n of Pascal's triangle.

Worked example. Expand $(1+2x)^5$ in ascending powers of x.

- Coefficients for $n=5$: $1, 5, 10, 10, 5, 1$:

$$\begin{aligned}(1+2x)^5 &= 1 + 5(2x) + 10(2x)^2 + 10(2x)^3 + 5(2x)^4 + (2x)^5\\ &= 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5\end{aligned}$$

- Check: $10(2x)^2 = 40x^2$ ✓ and $5(2x)^4 = 80x^4$ ✓.

Worked example – match a coefficient. In the expansion of $(2+kx)(1-3x)^5$, the coefficient of x^2 is -45. Find k.

- First $(1-3x)^5 = 1 - 15x + 90x^2 - \cdots$ (coefficients $1, 5, 10$: $5(-3x) = -15x$, $10(9x^2) = 90x^2$).
- The x^2 term of the product: $2 \times 90x^2 + kx \times (-15x) = (180 - 15k)x^2$:

$$180 - 15k = -45 \implies 15k = 225 \implies k = 15$$

- Check: $180 - 15(15) = 180 - 225 = -45$ ✓.

2 Practice

2.1 Expand $(1+3x)^4$ in ascending powers of x. [4]

2.2 Write down the first three terms of $(2+x)^5$ in ascending powers of x. [3]

2.3 Find the coefficient of x^2 in $(1-2x)^6$. [2]

2.4 Find the coefficient of x^3 in $(3+x)^5$. [3]

2.5 Find the coefficient of x^2 in $\left(1+\frac{x}{2}\right)^8$. [3]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q1.)*

(a) Find the first four terms, in ascending powers of x, of the binomial expansion of $(1-3x)^5$, giving each term in simplest form.

(b) In the expansion of $(2+15x)(1-3x)^5$, the coefficient of the term in x^2 is -45. Use the answer to part (a) to verify this. [6]

3.2 Use the first three terms of the expansion of $(1+2x)^6$ to estimate $(1.02)^6$, and compare your estimate with the exact value. [5]

Solutions

2.1 $1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4 = 1 + 12x + 54x^2 + 108x^3 + 81x^4$.

2.2 $32 + 5(16)x + 10(8)x^2 = 32 + 80x + 80x^2$.

2.3 $\binom{6}{2}(-2x)^2 = 15 \times 4x^2 = 60x^2$, so the coefficient is 60.

2.4 $\binom{5}{3}(3)^2(x)^3 = 10 \times 9 = 90$.

2.5 $\binom{8}{2}\left(\frac{x}{2}\right)^2 = 28 \times \frac{x^2}{4} = 7x^2$, coefficient 7.

3.1 (a) Coefficients $1, 5, 10, 10$:

$$(1 - 3x)^5 = 1 - 15x + 90x^2 - 270x^3 + \cdots$$

(b) The x^2 contributions: $2(90x^2) + 15x(-15x) = 180x^2 - 225x^2 = -45x^2$ ✓ – the stated coefficient is correct.

3.2 Expand first: $(1 + 2x)^6 = 1 + 6(2x) + 15(2x)^2 + \cdots = 1 + 12x + 60x^2 + \cdots$. For $(1.02)^6$ set $1 + 2x = 1.02$, so $x = 0.01$:

$$1 + 12(0.01) + 60(0.01)^2 = 1 + 0.12 + 0.006 = 1.126$$

Exact: $1.02^6 = 1.126162\ldots$ – the three-term estimate agrees to 3 decimal places ✓. (Take care to match x to the bracket: here $2x = 0.02$, not $x = 0.02$.)

Local review guidance: correct binomial coefficient with the right power of bx; simplified terms; coefficient equation set up from both contributions, or the substitution matched to the bracket. This is authored classroom guidance.