# 2.4 Sequences and series

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

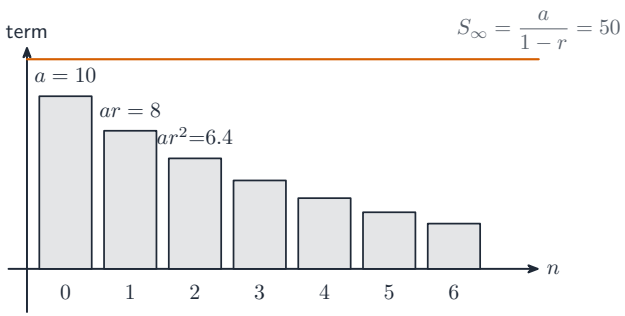
**KEY WORDS** arithmetic sequence • geometric sequence • sum to infinity • periodic • increasing

## Objective

Work with sequences given by formulas or by $x_{n+1} = f(x_n)$, arithmetic and geometric series (including $S_n$, $S_\infty$, $\Sigma$ notation and log-based term finding), and classify sequences as increasing, decreasing or periodic. Original teaching questions for specification items P2.4.1 to P2.4.4; the exam-style part adapts reviewed WMA12 June 2025 demand.

## 1 Key knowledge and worked examples

- **Arithmetic** : $a_n = a+(n-1)d$; $S_n = \frac{n}{2}\{2a+(n-1)d\}$; $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$. The sum formula's proof (add $S_n$ written forwards and backwards) should be known.
- **Geometric** : $a_n = ar^{n-1}$; $S_n = \dfrac{a(1-r^n)}{1-r}$; $S_\infty = \dfrac{a}{1-r}$ when $|r| < 1$.
- Classify from the terms: **increasing** , **decreasing** , **periodic** (the sequence repeats with a fixed period).




Each bar is $r = 0.8$ times the one before. With $|r| < 1$ the bars shrink fast enough that the total settles at a finite limit.

*$|r| < 1$ shrinks the terms; the total settles at $S\infty$.*

**Worked example – arithmetic with negative difference.** An arithmetic series starts $12, 9, 6, \ldots$ Find the 20th term and $S_{20}$.

- $a = 12$, $d = -3$:

$$a_{20} = 12 + 19(-3) = -45 \qquad S_{20} = \tfrac{20}{2}\{24 + 19(-3)\} = 10(-33) = -330$$

- Check: the terms fall by 3 each step; the sum of a series heading negative is negative ✓.

**Worked example – logs to find a term.** A geometric series has $a = 48$ and $r = \frac{1}{4}$. Find the smallest $n$ with $a_n < 0.001$.

$$48\left(\tfrac{1}{4}\right)^{n-1} < 0.001 \implies \left(\tfrac{1}{4}\right)^{n-1} < \frac{1}{48\,000}$$

$$(n-1)\log\tfrac{1}{4} < -\log 48\,000 \implies n-1 > \frac{\log 48\,000}{\log 4} = 7.57\ldots \implies n = 9$$

- Dividing by $\log\frac{1}{4}$ (negative) flips the inequality. Check: $n = 9$ gives $48/4^8 = 0.000732 < 0.001$ ✓; $n = 8$ gives $48/4^7 = 0.00293 > 0.001$ ✓ (correctly excluded).

## 2 Practice

**2.1** For $u_n = 3n - 2$, write down the 10th term. [1]

**2.2** For the arithmetic series $7 + 11 + 15 + \cdots$, find $S_{15}$. [3]

**2.3** A geometric sequence begins $8, 12, 18, \ldots$ Find its 10th term to 3 significant figures. [3]

**2.4** A geometric series has $S_\infty = 60$ and $r = \frac{1}{2}$. Find $a$. [2]

**2.5** The sequence defined by $x_{n+1} = 2x_n - 1$ with $x_1 = 3$: write down the first four terms and classify the sequence. [2]

**2.6** The sequence with $x_{n+1} = -x_n$ and $x_1 = 4$: write down the first four terms and state the period. [2]

## 3 Exam-style questions

**3.1** *(Adapted from Edexcel IAL WMA12 June 2025 Q7.)* A geometric series begins $18 + 12 + 8 + \cdots$

(a) Find the sum to infinity of the series.

(b) Given that the $k$th term of the series is less than 0.01, use algebra to find the smallest possible value of $k$. [5]

**3.2** *(Adapted from the same question.)* An arithmetic series has first term 500 and common difference $-7$. Find the greatest possible value of $S_n$. [5]

## Solutions

**2.1** $u_{10} = 3(10) - 2 = 28$.

**2.2** $a = 7$, $d = 4$: $S_{15} = \frac{15}{2}\{14 + 14 \times 4\} = 7.5 \times 70 = 525$.

**2.3** $r = \frac{12}{8} = 1.5$: $a_{10} = 8(1.5)^9 = 8 \times 38.44 = 308$ (3 s.f.).

**2.4** $a = S_\infty(1 - r) = 60 \times \frac{1}{2} = 30$.

**2.5** $3, 5, 9, 17$: increasing (each term exceeds the last).

**2.6** $4, -4, 4, -4$: periodic with period 2.

**3.1** (a) $r = \frac{2}{3}$, $|r| < 1$: $S_\infty = \dfrac{18}{1 - 2/3} = 54$. (b) $18\left(\frac{2}{3}\right)^{k-1} < 0.01 \implies \left(\frac{2}{3}\right)^{k-1} < \frac{1}{1800}$. Taking logs and dividing by the negative $\log\frac{2}{3}$ flips the sign:

$$k - 1 > \frac{\log(1/1800)}{\log(2/3)} = 18.48\ldots \implies k = 20$$

Check: $18\left(\frac{2}{3}\right)^{19} = 0.0081 < 0.01$ ✓ and $18\left(\frac{2}{3}\right)^{18} = 0.0122 > 0.01$ ✓ (so 19 is excluded).

**3.2** Terms stay positive while $500 - 7(n - 1) > 0$, i.e. $n \leq 72$; check the boundary: $u_{72} = 500 - 7(71) = 3 > 0$ and $u_{73} = -4 < 0$, so the maximum partial sum is

$$S_{72} = \tfrac{72}{2}\{1000 + 71(-7)\} = 36 \times 503 = 18\,108$$

Adding the negative 73rd term would reduce the total ✓.

Local review guidance: correct formula with $a$ and $r$ or $d$; exact or correctly rounded value; inequality direction preserved through negative-log division, or the boundary term checked. This is authored classroom guidance.