2.3 Coordinate geometry of the circle

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

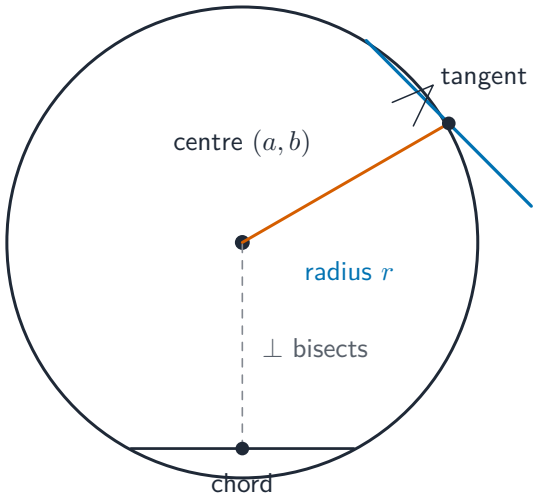
KEY WORDS chord • tangent • diameter • midpoint

Objective

Use the circle equation $(x-a)^2+(y-b)^2=r^2$ in both directions, complete the square on the expanded form, and apply the three examined circle properties. Original teaching questions for specification item P2.3.1; the exam-style part adapts reviewed WMA12 June 2025 demand.

1 Key knowledge and worked examples

- Centre (a,b), radius r: $(x-a)^2+(y-b)^2=r^2$.
- Expanded form $x^2+y^2+fx+gy+c=0$: complete both squares – centre $\left(-\frac{f}{2},-\frac{g}{2}\right)$, $r^2=\frac{f^2+g^2}{4}-c$.
- A **diameter** gives the centre as its **midpoint** and r^2 as a quarter of the diameter length squared.
- Properties: the angle in a semicircle is a right angle; the perpendicular from the centre to a **chord** bisects it; a **tangent** is perpendicular to the radius at the contact point.



Tangent $\perp$ radius at the contact point; the perpendicular from the centre to a chord **bisects** it; the angle in a semicircle is 90.

Tangent $\perp$ radius; centre-to-chord perpendicular bisects the chord.

Worked example – expanded form. Find the centre and radius of $x^2 + y^2 - 6x + 4y - 12 = 0$.

- Group and complete the square in x and in y:

$$(x^2 - 6x) + (y^2 + 4y) = 12 \implies (x-3)^2 - 9 + (y+2)^2 - 4 = 12$$

$$(x-3)^2 + (y+2)^2 = 25$$

- Centre $(3, -2)$, radius 5. Check with the point r right of the centre, $(8, -2)$: $64 + 4 - 48 - 8 - 12 = 0$ ✓, so it lies on the circle.

2 Practice

2.1 Find the centre and radius of $x^2 + y^2 + 8x - 2y + 8 = 0$. [3]

2.2 Write the equation of the circle with centre $(2, -5)$ and radius $\sqrt{13}$. [2]

2.3 Determine whether $(5, 2)$ lies on the circle $(x-1)^2 + (y-3)^2 = 17$. [2]

2.4 The points $(-3, 1)$ and $(5, 7)$ are the ends of a diameter. Find the circle's equation. [4]

2.5 The circle $x^2 + y^2 = 10$ passes through $(3, 1)$. Find the equation of the tangent at that point. [4]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q2.)* The line joining $(-1, -3)$ and $(7, 1)$ is the diameter of circle C.

(a) Find an equation for C.

(b) Hence find the exact coordinates of the point on C that is nearest the x-axis. [7]

3.2 The circle $(x-4)^2+(y-2)^2=25$ meets the line $x=0$ at $A(0,5)$ and $B(0,-1)$. Verify that the perpendicular from the centre to the chord AB bisects AB. [3]

Solutions

2.1 $(x+4)^2+(y-1)^2=9$: centre $(-4,1)$, radius 3.

2.2 $(x-2)^2+(y+5)^2=13$.

2.3 $(5-1)^2+(2-3)^2=16+1=17$ – yes, the point lies on the circle.

2.4 Centre: midpoint $=(1,4)$. Diameter squared: $8^2+6^2=100$, so $r^2=25$.

$$(x-1)^2+(y-4)^2=25$$

Check both endpoints: $16+9=25$ ✓ twice.

2.5 The radius to $(3,1)$ has gradient $\frac{1}{3}$; the tangent's gradient is -3:

$$y-1=-3(x-3) \implies y=-3x+10$$

Check: the tangent passes $(3,1)$ ✓, and $-3\times\frac{1}{3}=-1$ (perpendicular) ✓.

3.1 (a) Centre: $\left(\frac{-1+7}{2},\frac{-3+1}{2}\right)=(3,-1)$. Diameter squared: $8^2+4^2=80$, so $r^2=20$:

$$(x-3)^2+(y+1)^2=20$$

Check an endpoint: $(-4)^2+2^2=20$ ✓. (b) Nearest the x-axis means straight below the centre (the centre is 1 unit below the axis, less than $r=\sqrt{20}\approx 4.47$, so the circle crosses the axis; the lowest point is r below the centre):

$$(3,\ -1-\sqrt{20})=(3,\ -1-2\sqrt{5})$$

3.2 The centre is $(4,2)$. The perpendicular from $(4,2)$ to the vertical chord $x=0$ is the horizontal line $y=2$, meeting AB at $(0,2)$. The midpoint of $A(0,5)$ and $B(0,-1)$ is $\left(0,\frac{5+(-1)}{2}\right)=(0,2)$ – the same point, so the perpendicular bisects the chord ✓.

Local review guidance: midpoint/completing-the-square method; exact centre, radius and equation; property applied with the right perpendicular. This is authored classroom guidance.