1.6 Differentiation and its applications

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS derivative • gradient of the tangent • rate of change • second derivative • normal

Objective

Differentiate x^n and sums, differences and constant multiples (expanding products and splitting quotients first), interpret the derivative as gradient and rate of change, and find tangents and normals. Original teaching questions for specification items P1.4.1 to P1.4.3; the exam-style part adapts reviewed WMA11 June 2025 demand.

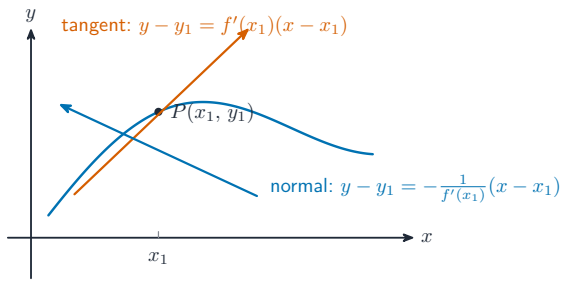
1 Key knowledge and worked examples

The **derivative** $f'(x) = \dfrac{dy}{dx}$ is the gradient of the tangent – the limit of chord gradients – and the **rate of change** of y with respect to x. Differentiate again for the **second derivative** $f''(x)$. The chain rule is not needed in P1.

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \frac{d}{dx}\left(4x^6\right) = 24x^5$$

Expand and split first – the specification expects both skills:

$$y = (2x+5)(x-1) = 2x^2+3x-5 \implies \frac{dy}{dx} = 4x+3 \qquad y = \frac{3x^2+5}{x^2} = 3+5x^{-2} \implies \frac{dy}{dx} = -10x^{-3}$$



Tangent gradient $= f'(x_1)$; *normal gradient* $= -1/f'(x_1)$.

Worked example – tangent and normal. The curve $y = x^2 - 3x$ passes through $x = 2$. Find the equations of the tangent and the normal there.

- $\dfrac{dy}{dx} = 2x - 3$, so at $x = 2$ the gradient is 1 and $y = 4 - 6 = -2$: point $(2, -2)$.
- Tangent: $y - (-2) = 1(x - 2) \implies y = x - 4$.
- Normal: gradient $-1 \div 1 = -1$: $y + 2 = -(x - 2) \implies y = -x$.
- Check: the normal passes $(2, -2)$: $-2 = -2$ ✓; tangent at $x = 2$: $2 - 4 = -2$ ✓.

Worked example – gradient condition. The gradient of the curve $y = ax^{3/2}$ at $x = 4$ is 12. Find a.

$$\frac{dy}{dx} = \tfrac{3}{2}ax^{1/2} = \tfrac{3}{2}a\sqrt{4} = 3a = 12 \implies a = 4$$

- Check: with $a = 4$, $y = 4x^{3/2}$ and $y'(4) = 6\sqrt{4} = 12$ ✓.

2 Practice

2.1 Differentiate $y = 4x^6$. [1]

2.2 Differentiate $y = x^{5/2} + 3x$. [2]

2.3 Expand and differentiate $y = (x + 2)(x - 3)$. [3]

2.4 Differentiate $y = \dfrac{2x^3 - x}{x}$. [3]

2.5 Find the gradient of $y = \sqrt{x}$ at $x = 9$. [2]

2.6 Find the second derivative of $y = 3x^4 - x^2$. [2]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA11 June 2025 Q2.)* The curve C has equation $y = 3x^{4/3} - 2x + 1$. The point P on C has x-coordinate 8. Find the gradient of C at P. [3]

3.2 *(Adapted from Edexcel IAL WMA11 June 2025 Q8.)* The curve $y = x^{3/2} + \dfrac{1}{x}$ passes through the point where $x = 1$. Find the equations of the tangent and the normal to the curve at that point. [6]

Solutions

2.1 $\dfrac{dy}{dx} = 24x^5$.

2.2 $\dfrac{dy}{dx} = \frac{5}{2}x^{3/2} + 3$.

2.3 $(x+2)(x-3) = x^2 - x - 6$, so $\dfrac{dy}{dx} = 2x - 1$.

2.4 Split first: $y = 2x^2 - 1$, so $\dfrac{dy}{dx} = 4x$.

2.5 $y = x^{1/2} \implies \dfrac{dy}{dx} = \frac{1}{2}x^{-1/2}$; at $x = 9$: $\frac{1}{2} \times \frac{1}{3} = \dfrac{1}{6}$.

2.6 $\dfrac{dy}{dx} = 12x^3 - 2x$; $\dfrac{d^2y}{dx^2} = 36x^2 - 2$.

3.1 Differentiate term by term:

$$\frac{dy}{dx} = 3 \times \tfrac{4}{3}x^{1/3} - 2 = 4x^{1/3} - 2$$

At $x = 8$: $x^{1/3} = 2$, so the gradient is $4(2) - 2 = \mathbf{6}$. Check: $8^{1/3} = 2$ because $2^3 = 8$ ✓.

3.2 At $x = 1$: $y = 1^{3/2} + 1 = 2$, so the point is $(1, 2)$.

$$\frac{dy}{dx} = \tfrac{3}{2}x^{1/2} - x^{-2} \qquad \text{so at } x = 1: \ \tfrac{3}{2} - 1 = \tfrac{1}{2}$$

- Tangent (gradient $\frac{1}{2}$): $y - 2 = \frac{1}{2}(x - 1) \implies y = \frac{1}{2}x + \frac{3}{2}$.
- Normal (gradient $-1 \div \frac{1}{2} = -2$): $y - 2 = -2(x - 1) \implies y = -2x + 4$.
- Check: tangent at $x = 1$ gives $y = 2$ ✓; normal at $x = 1$ gives $y = 2$ ✓; the gradients multiply to $\frac{1}{2} \times (-2) = -1$ ✓.

Local review guidance: power reduced correctly after any expansion; exact gradient or equation; negative reciprocal used for the normal. This is authored classroom guidance.