1.2 Quadratics, the discriminant and completing the square

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS discriminant • quadratic formula • completing the square • repeated root

Objective

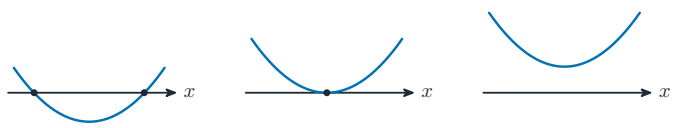
Sketch quadratic functions, use the discriminant $b^2 - 4ac$ to count roots, solve quadratics by factorisation, the formula and completing the square, and factorise cubics. Original teaching questions for specification items P1.1.3, P1.1.4, P1.1.5 and P1.1.10; the exam-style part adapts reviewed WMA11 June 2025 demand.

1 Key knowledge and worked examples

For $ax^2 + bx + c = 0$:

- **discriminant** $\Delta = b^2 - 4ac$: $\Delta > 0$ two distinct real roots; $\Delta = 0$ one repeated root; $\Delta < 0$ no real roots.
- **quadratic formula**: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- **completing the square** : $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 + c - \frac{b^2}{4}$; the turning point is $\left(-\frac{b}{2},\, c - \frac{b^2}{4}\right)$.

Cubics factorise by spotting a common factor or a root: $x^3+4x^2+3x = x(x^2+4x+3) = x(x+1)(x+3)$.



$b^2 - 4ac > 0$ two distinct roots | $b^2 - 4ac = 0$ one repeated root | $b^2 - 4ac < 0$ no real roots

The discriminant counts intersections with the x-axis.

Worked example – formula. Solve $2x^2 - 5x + 1 = 0$, giving exact answers.

- Known: $a = 2$, $b = -5$, $c = 1$. Why the formula: the expression does not factorise.

$$x = \frac{5 \pm \sqrt{25 - 8}}{4} = \frac{5 \pm \sqrt{17}}{4}$$

- Check: $\sqrt{17} \approx 4.123$, so $x \approx 2.281$ or 0.219; substituting 0.219: $2(0.048) - 5(0.219) + 1 \approx 0$ ✓.

Worked example – equal roots condition. The equation $kx^2 + 4x + (k - 2) = 0$ has equal roots. Find the possible values of k.

- Equal roots means $\Delta = 0$:

$$b^2 - 4ac = 16 - 4k(k - 2) = -4k^2 + 8k + 16 = 0$$

- Divide by -4: $k^2 - 2k - 4 = 0$, so

$$k = \frac{2 \pm \sqrt{4 + 16}}{2} = 1 \pm \sqrt{5}$$

- Check: for $k = 1 + \sqrt{5} \approx 3.236$: $16 - 4(3.236)(1.236) = 16 - 16.0 = 0$ ✓.

2 Practice

2.1 Factorise $x^2 - 7x + 12$. [2]

2.2 Write $x^2 - 8x + 11$ in the form $(x + p)^2 + q$. [3]

2.3 Solve $3x^2 + 2x - 1 = 0$. [3]

2.4 Show that $2x^2 + 3x + 5 = 0$ has no real roots. [2]

2.5 Factorise fully $x^3 + 5x^2 + 6x$. [3]

2.6 The equation $x^2 + kx + 9 = 0$ has equal roots. Find the possible values of k. [2]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA11 June 2025 Q10.)* Solve the equation $x^8 - 5x^4 + 4 = 0$ by treating it as a quadratic in x^4. Give all real solutions. [4]

3.2 *(Adapted from Edexcel IAL WMA11 June 2025 Q10(b).)* Find the set of values of k for which $(k + 2)x^2 + 6x + k = 0$ has no real roots. [4]

Solutions

2.1 $(x-3)(x-4)$: $(-3)(-4) = 12$ and $-3-4 = -7$ ✓.

2.2 $p = -8/2 = -4$, and $q = 11 - 16 = -5$: $(x-4)^2 - 5$.

2.3 Factorise: $(3x-1)(x+1) = 0$, so $x = \frac{1}{3}$ or $x = -1$.

2.4 $\Delta = 9 - 4(2)(5) = 9 - 40 = -31 < 0$, so there are no real roots. A negative discriminant is the complete justification – no further working needed.

2.5 Take out x: $x(x^2 + 5x + 6) = x(x+2)(x+3)$.

2.6 $\Delta = k^2 - 36 = 0$, so $k = \pm 6$.

3.1 Substitute $z = x^4$: $z^2 - 5z + 4 = (z-1)(z-4) = 0$, so $x^4 = 1$ or $x^4 = 4$.

- $x^4 = 1 \implies x^2 = 1 \implies x = \pm 1$.
- $x^4 = 4 \implies x^2 = 2$ (only the positive value is real) $\implies x = \pm\sqrt{2}$. All four real solutions: $x = 1$, $x = -1$, $x = \sqrt{2}$, $x = -\sqrt{2}$. Check $x = \sqrt{2}$: $x^4 = 4$, so $x^8 - 5x^4 + 4 = 16 - 20 + 4 = 0$ ✓.

3.2 No real roots needs $b^2 - 4ac < 0$ with $a = k+2$, $b = 6$, $c = k$:

$$36 - 4(k+2)k = 36 - 4k^2 - 8k < 0$$

Divide by -4 (flip the sign): $k^2 + 2k - 9 > 0$. Roots of $k^2 + 2k - 9 = 0$:

$$k = \frac{-2 \pm \sqrt{4+36}}{2} = -1 \pm \sqrt{10}$$

The parabola in k opens upward, so the inequality holds outside the roots: $k < -1 - \sqrt{10}$ or $k > -1 + \sqrt{10}$. Check $k = 0$ (inside): $36 > 0$ – two real roots, correctly excluded ✓.

Local review guidance: correct discriminant or substitution set up; exact values; the correct region or root count with a reason. This is authored classroom guidance.