1.2.2 Graphs, sketching and transformations

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

KEY WORDS cubic • reciprocal function • asymptote • transformation

Objective

Sketch cubics, reciprocal curves and their asymptotes, read intersections as solutions, and apply single transformations to graphs. Original teaching questions for specification items P1.1.11 and P1.1.12; the exam-style part adapts reviewed WMA11 June 2025 demand.

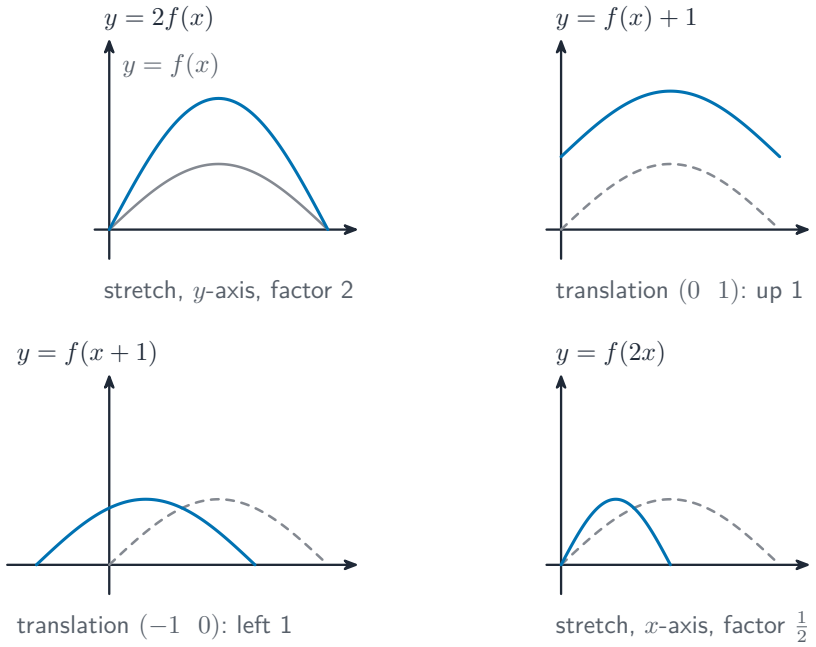
1 Key knowledge and worked examples

Sketch these families (sketch means shape + key points, not a plotted table):

- **cubics** such as $y = (x-1)(x+2)(x-3)$: mark the three x-intercepts and the y-intercept $(0,\ (0-1)(0+2)(0-3)) = (0, 6)$.
- **reciprocal functions** $y = \dfrac{k}{x}$ and $y = \dfrac{k}{x^2}$ $(x \neq 0)$: both branches live either side of an **asymptote** – the y-axis for these.
- Intersections answer equations: $y = f(x)$ meets $y = g(x)$ where $f(x) = g(x)$.

One **transformation** at a time on $y = f(x)$:

New equation	Effect
$y = af(x)$	stretch parallel to the y-axis, scale factor a
$y = f(x) + a$	translation $\begin{pmatrix} 0 \\ a \end{pmatrix}$
$y = f(x + a)$	translation $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
$y = f(ax)$	stretch parallel to the x-axis, scale factor $\dfrac{1}{a}$



Name direction, type and scale factor (or vector).

Worked example. The graph of $y = x^2$ is transformed to the graph of $y = (x+5)^2$. Describe the transformation, and state the coordinates of the new turning point.

- Write $(x+5)^2 = f(x+5)$ with $f(x) = x^2$: this is a **translation by** $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$.
- The turning point $(0, 0)$ moves to $(-5, 0)$.
- Check: $(-5+5)^2 = 0$ ✓ – the new curve touches the x-axis at -5.

2 Practice

2.1 Sketch $y = \dfrac{1}{x}$, labelling its asymptotes. [2]

2.2 Sketch $y = x^3 - x$, marking the intercepts with the axes. [3]

2.3 Sketch $y = \dfrac{2}{x^2}$ for $x \neq 0$, labelling its asymptotes. [2]

2.4 Describe the single transformation mapping $y = \sin x$ onto $y = 3\sin x$. [2]

2.5 Describe the single transformation mapping $y = \sqrt{x}$ onto $y = \sqrt{x+4}$, and state where the transformed curve meets the x-axis. [3]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA11 June 2025 Q4.)* The curve C has equation $y = 5\sin x$, with x in degrees.

(a) State the period of C.

(b) State the number of solutions of $5\sin x = -5$ in the interval $0 \leq x \leq 720°$, justifying your answer. [3]

3.2 *(Adapted from Edexcel IAL WMA11 June 2025 Q9.)*

(a) Describe fully the single transformation that maps the graph of $y = \sqrt{x}$ onto the graph of C_1: $y = \sqrt{3x}$.

(b) The curves C_1: $y = \sqrt{3x}$ and C_2: $y = 2\sqrt{x}$ intersect at P. Use algebra to find the exact coordinates of P. [6]

Solutions

2.1 Two branches in quadrants 1 and 3, each approaching the x-axis and y-axis asymptotes (the axes themselves); the curve passes through $(1, 1)$ and $(-1, -1)$.

2.2 $x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$: the curve crosses at $x = -1, 0, 1$ and passes through the origin with y-intercept 0.

2.3 Both branches above the x-axis (squares are positive), symmetric in the y-axis, with the x-axis and y-axis as asymptotes; through $(\pm 1, 2)$.

2.4 A stretch parallel to the y-axis with scale factor 3 (amplitude 1 becomes 3).

2.5 $y = \sqrt{x + 4} = f(x + 4)$: a translation by $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$. The curve meets the x-axis where $x + 4 = 0$, at $(-4, 0)$.

3.1 (a) $360°$ (the transformation from $y = \sin x$ is a y-stretch, which does not change the period). (b) $5 \sin x = -5 \implies \sin x = -1 \implies x = 270° + 360°n$. In $0 \le x \le 720°$: $x = 270°$ and $x = 630°$ – **2 solutions**. Check: $\sin 270° = \sin 630° = -1$ ✓.

3.2 (a) $y = \sqrt{3x} = f(3x)$: a **stretch parallel to the x-axis with scale factor $\frac{1}{3}$**. (b) Set the y-values equal and square both sides (both are non-negative for $x \ge 0$):

$$\sqrt{3x} = 2\sqrt{x} \implies 3x = 4x \implies x = 0$$

The only intersection is $P = (0, 0)$: check $\sqrt{3 \times 0} = 0 = 2\sqrt{0}$ ✓. For $x > 0$, $\sqrt{3x} = \sqrt{3}\sqrt{x}$ and $\sqrt{3} < 2$, so C_1 stays below C_2 – the curves never cross again.

Local review guidance: correct transformation named with direction and factor; exact intersection algebra with both sides' domains respected; justification from the sketch. This is authored classroom guidance.