Exercise walk-through

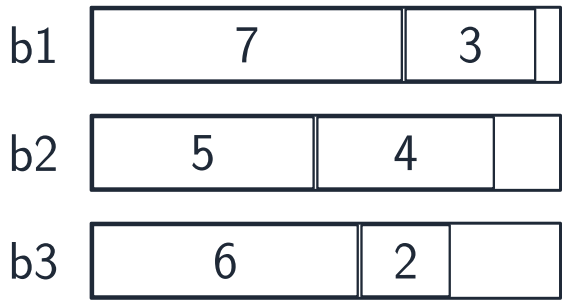
9.4
Critical path analysis

Pearson Edexcel · International A-Level · Mathematics

Critical path analysis (2)

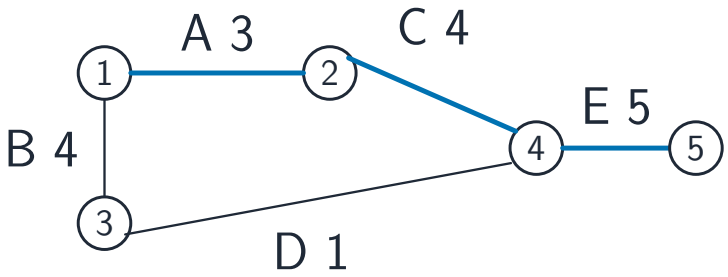
- A **critical path** is a path of activities with zero **total float**. Its length is the shortest time in which the project can be completed.
- Earliest times: A ends at 3, B at 4, C at 7, D at 5. E starts at $\max(7, 5) = 7$ and ends at 12.
- The **critical path** is A–C–E, and the project takes 12 days.
- D must finish by 7, so its total float is $7 - 4 - 1 = 2$ days. B must finish by 6, so its float is $6 - 0 - 4 = 2$ days.

Figure: Critical path analysis



capacity 10

first-fit: (7,3), (5,4), (6,2) — three bins, none over capacity



event times 0; 3 / 4; 7; 12

critical path A–C–E = 12 days; D has float 2

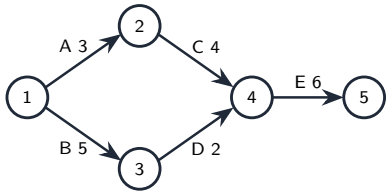
- Left: the first-fit bins of worked example 3. Right: the activity network of worked example 8, with the **critical path** A–C–E highlighted.
- Match each part of the diagram to the text.

Key terms

- **Minimum spanning tree**: a tree connecting all the vertices with the least total weight.
- **Dijkstra's algorithm**: an algorithm for the shortest path from one vertex to the others.
- **Route inspection problem**: finding the shortest closed route that uses every edge.

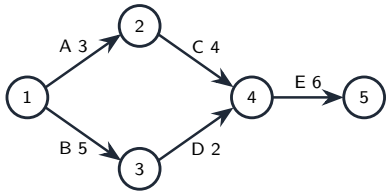
Exercise 2.1

2.1 Activities A (2), B (5), C (3 after A), D (1 after A and B). Draw the precedence table as an activity network description (list arcs into and out of each event) and find the project duration. [4]



Exercise 2.1

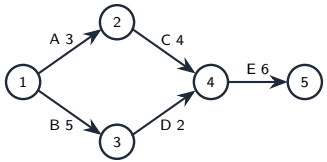
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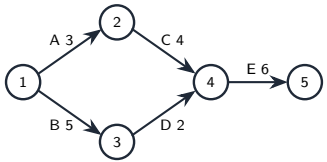
2.1 Events: start s; A: s→e1 (2), B: s→e2 (5); C: e1→e3 (3); D: e1→e3 and e2→e3 – D depends on both A and B, so e1 must connect to e3 and e2 to e3 (a dummy from e2 to e1 if drawn with D leaving a shared node). Durations: A–C = 2 + 3 = 5; B = 5; D needs both done: D's earliest start = max(2, 5) = 5. Project duration = max(5, 5 + 1) = 6 days.

Exercises 2.2·2.3

2.2 For a project with earliest event times 0, 4, 7, 9 and latest times 0, 4, 7, 9 along one chain, state the float of each activity on the chain. [2]

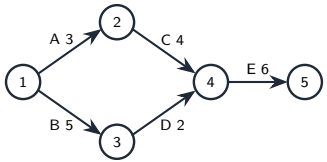


2.3 An activity lasts 6 days, its earliest start is day 5 and its latest finish is day 16. Find its total float. [2]



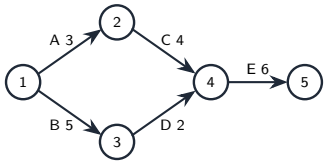
Exercises 2.2·2.3

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2.2 Earliest = latest at every event: every activity on the chain is critical, float 0.

2.3 An activity lasts 6 days, its earliest start is day 5 and its latest finish is day 16. Find its total float. [2]

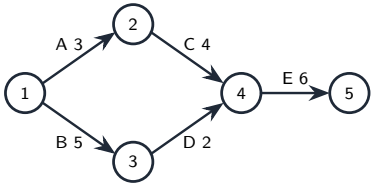


2.3 Float $= 16 - 5 - 6 = 5$ days.

Exercise 3.1

3.1 *(Original.)* A project has activities A (4), B (3), C (6 after A), D (2 after A and B), E (5 after C and D), F (3 after D).

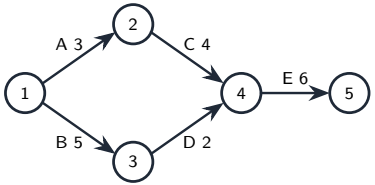
(a) Draw an activity network for the project (from the precedence table), using a dummy where necessary, and describe where the dummy is needed. (b) Carry out a forward pass and a backward pass, listing the earliest and latest times for each event. (c) Find the critical path and the duration of the project. (d) Find the total float of F. [11]



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3.1 (a) Arcs: A from start to event 2; B from start to event 3; C from event 2 to event 4; D depends on A and B, so a dummy (0) runs from event 2 to event 3 and D leaves event 3 – but D must end at its OWN event 5 (time 4 + 2 = 6), since F follows D alone and must not inherit C's dependency. A dummy from event 5 to event 4 carries D into E's

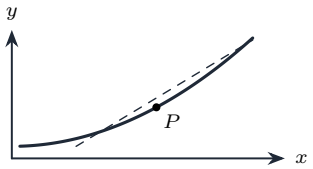
Full solution on next slide

Full solution 3.1

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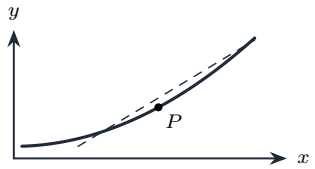
Exercise 3.2

3.2 *(Original.)* Using the project of 3.1: (a) draw a Gantt chart for the project, labelling the critical activities; (b) state the minimum number of workers needed to avoid delaying any critical activity; (c) explain why F can start as late as day 12. [6]



Exercise 3.2

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3.2 (a) Bars: A from 0 for 4; B from 0 for 3; C from 4 for 6; D from 4 for 2; E from 10 for 5; F from 6 for 3 with its slack after it. Critical activities A, C, E on one unbroken chain. (b) One worker suffices for the critical chain if nothing else overlaps it in time – but B (0–3) and D (4–6) overlap A (0–4) and C (4–10): at least 2 workers are needed. (c) F's latest finish is 15 and it lasts 3 days, so it can start as late as $15 - 3 = 12$ (its earliest start is 6, giving the 6-day float).

Local review guidance: dummies carry exactly the dependency that shared endpoints cannot; forward pass takes maxima, backward pass minima; float from latest finish − earliest start − duration; Gantt bars from earliest starts with slack trailing. This is authored classroom guidance.

Exercise 3.3 (a)

3.3 The activity network shows the activities of a project and their durations in days.

(a) Carry out a forward pass and a backward pass to find the early and late event times. [4]

Exercise 3.3 (a)

3.3 The activity network shows the activities of a project and their durations in days.

(a) Carry out a forward pass and a backward pass to find the early and late event times. [4]

(a)

- forward pass, the largest of the incoming totals: $e_1 = 0$, $e_2 = 3$, $e_3 = \max(5,\ 3+2) = 5$, $e_4 = 3 + 4 = 7$, $e_5 = \max(5 + 6,\ 7 + 3) = 11$, $e_6 = \max(7 + 4,\ 11 + 2) = 13$
- backward pass, the smallest of the outgoing differences: $l_6 = 13$, $l_5 = 11$, $l_4 = \min(11 - 3,\ 13 - 4) = 8$, $l_3 = 5$, $l_2 = \min(8 - 4,\ 5 - 2) = 3$, $l_1 = 0$

Exercise 3.3 (b)

3.3 The activity network shows the activities of a project and their durations in days.

(b) State the critical activities and the length of the project. [3]

Exercise 3.3 (b)

3.3 The activity network shows the activities of a project and their durations in days.

(b) State the critical activities and the length of the project. [3]

(b)

- events 1, 2, 3, 5, 6 have $e = l$, so the zero-float activities are **A, B, D, E, H** (paths A–D–E–H and B–E–H); project length **13 days**

Exercise 3.3 (c)

3.3 The activity network shows the activities of a project and their durations in days.

(c) Find the total float on each of activities C, F and G. [3]

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(c)

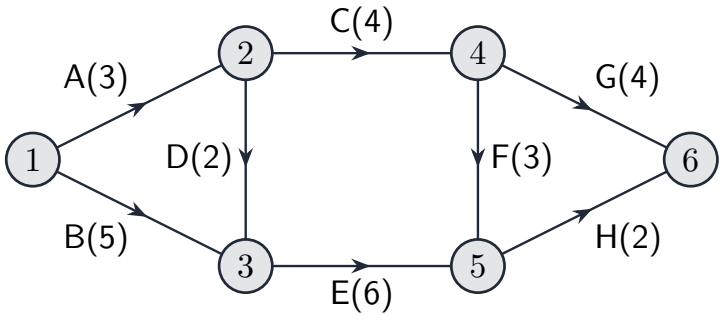
- total float = late time of the end event − early time of the start event − duration:

  $\text{float}(C) = 8 - 3 - 4 = \mathbf{1}, \quad \text{float}(F) = 11 - 7 - 3 = \mathbf{1}, \quad \text{float}(G) = 13 - 7 - 4 = \mathbf{2}$

Exercise 3.3 (d)

3.3 The activity network shows the activities of a project and their durations in days.

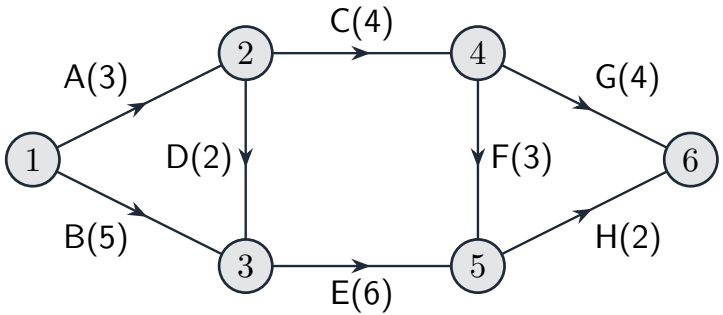
(d) Find a lower bound for the number of workers needed to finish the project in the minimum time. [2]



Exercise 3.3 (d)

3.3 The activity network shows the activities of a project and their durations in days.

(d) Find a lower bound for the number of workers needed to finish the project in the minimum time. [2]



(d)

- lower bound = total activity time ÷ project length = $\frac{29}{13}$ = 2.23, rounded up to **3** workers