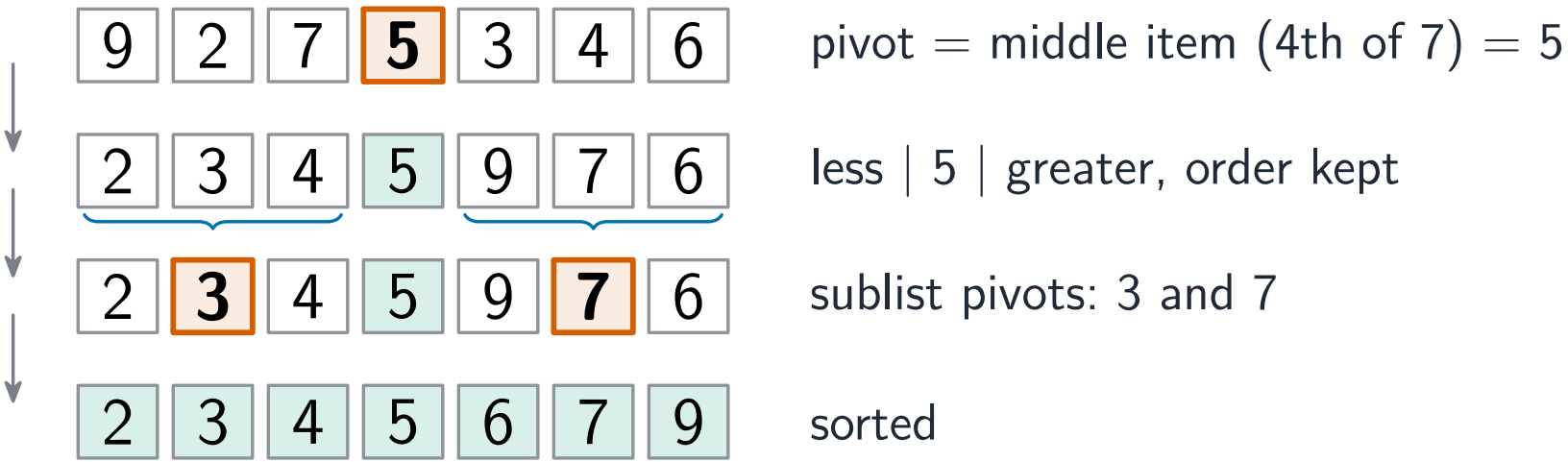
Exercise walk-through

9.1
Sorting and packing algorithms

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Figure: Sorting and packing algorithms

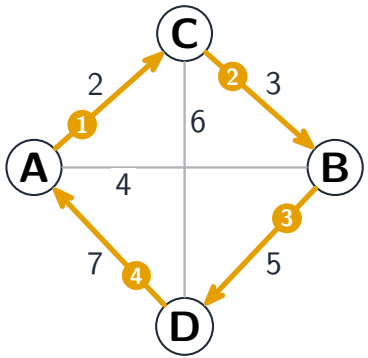


- **Quick sort** with the middle-item pivot: quote the pivot and the list after every partition
- Match each part of the diagram to the text.

The travelling salesman problem

- **Upper bound**: an actual route. Doubling the weight of the MST gives one; shortcuts improve it. The **nearest neighbour algorithm** also gives one: from the start, go to the nearest unvisited vertex each time, then return to the start.
- **Lower bound**: delete one vertex, find the MST of the rest, and add the two smallest edges from the deleted vertex.
- Nearest neighbour from A: A–C (2), C–B (3), B–D (5), D–A (7). Total 17, an upper bound.

Figure: The travelling salesman problem



nearest neighbour from A:
A–C–B–D–A $= 2+3+5+7 = 17$
upper bound

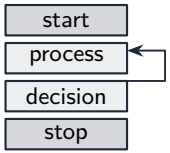
delete A: MST of B, C, D
$= 3+5 = 8$
plus AC + AB $= 2+4$:
lower bound 14

$14 \leq$ optimal tour ≤ 17

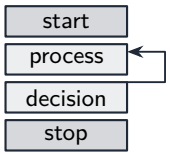
- Nearest neighbour gives an upper bound, the deleted-vertex MST a lower bound: the optimal tour lies between 14 and 17
- Match each part of the diagram to the text.

Exercises 2.1·2.2

2.1 Bubble sort 5, 1, 4, 2, showing the list after each pass. [3]

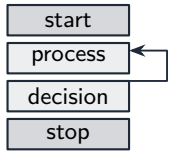


2.2 Quick sort 1, 6, 3, 5, 7, 2, 9, stating each pivot and the list after each partition. [4]



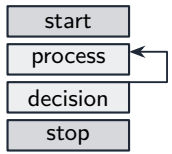
Exercises 2.1·2.2

2.1 Bubble sort 5, 1, 4, 2, showing the list after each pass. [3]



2.1 Pass 1: 5>1 swap, 5>4 swap, 5>2 swap → 1, 4, 2, 5. Pass 2: 4>2 swap → 1, 2, 4, 5. Pass 3: no swaps → sorted, stop. Final: 1, 2, 4, 5.

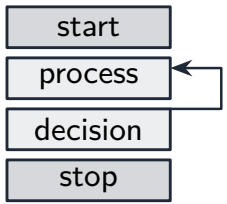
2.2 Quick sort 1, 6, 3, 5, 7, 2, 9, stating each pivot and the list after each partition. [4]



2.2 Pivot (position 4 of 7) = 5: less = 1, 3, 2; greater = 6, 7, 9, giving 1, 3, 2, **5**, 6, 7, 9. Sublist 1, 3, 2 (pivot 3): items less than 3, in order, are 1, 2 and none are greater, so the sublist becomes 1, 2, **3**; the pair 1, 2 (pivot 1) gives **1**, 2. Sublist 6, 7, 9 (pivot 7): less = 6, greater = 9 – already sorted. Final list: 1, 2, 3, 5, 6, 7, 9.

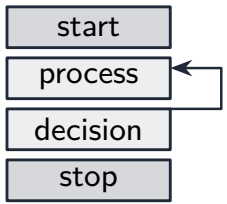
Exercise 2.3

2.3 Bin capacity 8; items 5, 3, 4, 2, 6, 1. Apply first-fit, showing each placement, and state the number of bins used. [3]



Exercise 2.3

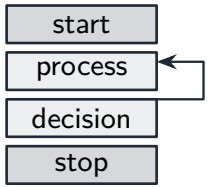
2.3 Bin capacity 8; items 5, 3, 4, 2, 6, 1. Apply first-fit, showing each placement, and state the number of bins used. [3]



2.3 5→b1, 3→b1 (8), 4→b2, 2→b2 (6), 6→b3, 1→b2 (7): bins (5,3), (4,2,1), (6) – 3 bins.

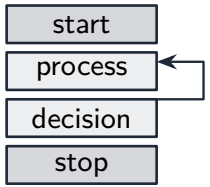
Exercise 3.1

3.1 *(Original.)* A courier has parcels of weights 9, 4, 6, 2, 8, 5, 3, 7 (kg) and boxes of capacity 15 kg. (a) Use the first-fit algorithm to pack the parcels, listing the contents of each box in order. (b) Use the first-fit decreasing algorithm to pack them. (c) Show that the total weight implies at least three boxes, and comment on the two algorithms' results. [9]



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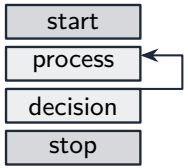


3.1 (a) First-fit (capacity 15): 9→b1 (9), 4→b1 (13), 6→b2 (6), 2→b1 (15 – exact fit), 8→b2 (14), 5→b3 (5), 3→b3 (8), 7→b3 (15): boxes (9,4,2), (6,8), (5,3,7) – 3 boxes. (b) Decreasing order 9, 8, 7, 6, 5, 4, 3, 2: 9→b1, 8→b2, 7→b3, 6→b1 (15), 5→b2 (13), 4→b3 (11), 3→b3 (14), 2→b2 (15): boxes (9,6), (8,5,2), (7,4,3) – 3 boxes. (c) Total weight $9+4+6+2+8+5+3+7=44$, and $\frac{44}{15}=2.93\ldots$ so at least 3 boxes; both algorithms achieve exactly 3 here – the lower bound is met, so both packings are optimal.

Exercise 3.2

3.2 *(Original.)* The list 1, 6, 3, 4, 8, 2, 9 is to be sorted into ascending order.

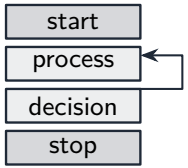
(a) Use the quick sort algorithm, stating the pivot at each stage and the list after each partition. (b) State one advantage quick sort has over bubble sort on large lists. [8]



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(a) Use the quick sort algorithm, stating the pivot at each stage and the list after each partition. (b) State one advantage quick sort has over bubble sort on large lists. [8]



3.2 (a) Pivot (position 4 of 7) = 4: less in order = 1, 3, 2; greater = 6, 8, 9, giving 1, 3, 2, **4**, 6, 8, 9. Sublist 1, 3, 2 (pivot 3): less = 1, 2; greater = none → 1, 2, **3**; then (1, 2) pivot 1: **1**, 2. Sublist 6, 8, 9 (pivot 8): less = 6; greater = 9 → 6, **8**, 9. Final: 1, 2, 3, 4, 6, 8, 9. (b) On large lists quick sort is generally much faster (fewer comparisons on average), because each partition splits the problem instead of moving items one position per comparison.

Local review guidance: pivots stated and partitions listed; placements shown item by item with running totals; lower-bound comparison quoted. This is authored classroom guidance.

Exercise 3.3 (a)

3.3 Eight boxes, of weights 12, 9, 8, 7, 6, 5, 4 and 3 kg, are to be packed into crates that each hold at most 20 kg. A student used the first-fit decreasing algorithm and produced the packing shown in the figure.

(a) Find a lower bound for the number of crates needed. [2]

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(a) Find a lower bound for the number of crates needed. [2]

(a)

- lower bound $= \dfrac{\text{total weight}}{\text{capacity}}$, rounded up: $\dfrac{12+9+8+7+6+5+4+3}{20} = \dfrac{54}{20} = 2.7$, so **3** crates

Exercise 3.3 (b)

3.3 Eight boxes, of weights 12, 9, 8, 7, 6, 5, 4 and 3 kg, are to be packed into crates that each hold at most 20 kg. A student used the first-fit decreasing algorithm and produced the packing shown in the figure.

(b) Explain the error the student made in applying the first-fit decreasing algorithm. [2]

Exercise 3.3 (b)

3.3 Eight boxes, of weights 12, 9, 8, 7, 6, 5, 4 and 3 kg, are to be packed into crates that each hold at most 20 kg. A student used the first-fit decreasing algorithm and produced the packing shown in the figure.

(b) Explain the error the student made in applying the first-fit decreasing algorithm. [2]

(b)

- when the 4 kg box was placed, crate 2 held $9 + 7 = 16$ kg; first fit puts each box in the first crate it fits, so it belonged in crate 2, not crate 3

Exercise 3.3 (c)

3.3 Eight boxes, of weights 12, 9, 8, 7, 6, 5, 4 and 3 kg, are to be packed into crates that each hold at most 20 kg. A student used the first-fit decreasing algorithm and produced the packing shown in the figure.

(c) Apply the first-fit decreasing algorithm correctly, showing which crate each box goes into. [3]

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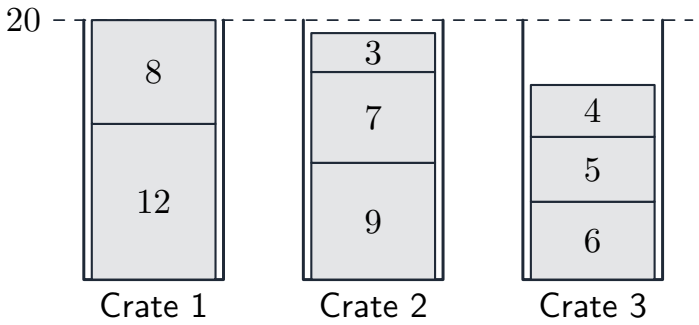
(c)

- the list is already in decreasing order: 12 → crate 1; 9 → crate 2; 8 → crate 1 (20); 7 → crate 2 (16); 6 → crate 3; 5 → crate 3 (11); 4 → crate 2 (20); 3 → crate 3 (14)
- crate 1: 12, 8; crate 2: 9, 7, 4; crate 3: 6, 5, 3

Exercise 3.3 (d)

3.3 Eight boxes, of weights 12, 9, 8, 7, 6, 5, 4 and 3 kg, are to be packed into crates that each hold at most 20 kg. A student used the first-fit decreasing algorithm and produced the packing shown in the figure.

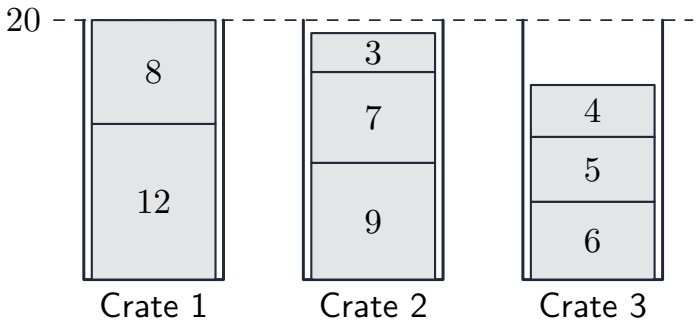
(d) State, with a reason, whether your packing in (c) is optimal. [1]



Exercise 3.3 (d)

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(d) State, with a reason, whether your packing in (c) is optimal. [1]



(d)

- it uses 3 crates, which equals the lower bound, so it is optimal

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