Exercise walk-through

7.4
Collisions and restitution

Pearson Edexcel · International A-Level · Mathematics

Centres of mass: particles, laminas and wire frameworks

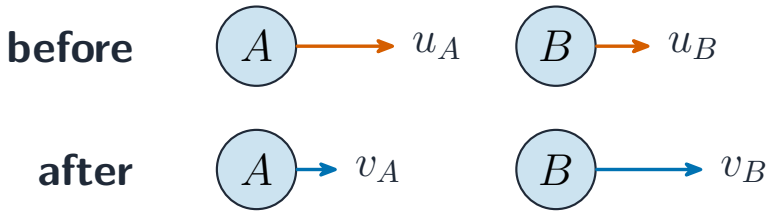
- For particles of masses $m_1, m_2, \ldots$ at $x_1, x_2, \ldots$, the **centre of mass** is at
- and in the same way for $\bar{y}$.
- For a uniform composite body, use a table:
- 1. split the body into pieces whose centres of mass you know (the booklet gives arcs, sectors, semicircles and triangles); 2. use area as the "mass" of a lamina, and length as the "mass" of a wire; 3. take moments about a convenient line.

$$\bar{x} = \frac{\sum mx}{\sum m}$$

Collisions and restitution (2)

- Solving: $v_P = \dfrac{(2-e)u}{3}$ and $v_Q = \dfrac{2(1+e)u}{3}$.
- The **kinetic energy** after is $\frac{1}{2}(2m)v_P^2 + \frac{1}{2}mv_Q^2 = \dfrac{mu^2}{3}(2+e^2)$. Before, it was mu^2, so the loss is $\dfrac{mu^2}{3}(1-e^2)$.
- If the impulse on Q is $\frac{8}{9}mu$, then $m \times \dfrac{2(1+e)u}{3} = \dfrac{8mu}{9}$, so $e = \frac{1}{3}$, $v_Q = \frac{8}{9}u$ and $v_P = \frac{5}{9}u$.

Figure: Collisions and restitution



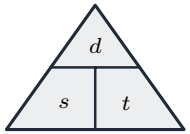
$$e = \frac{\text{separation speed}}{\text{approach speed}} = \frac{v_B - v_A}{u_A - u_B}$$

$e = 1$ elastic; $e = 0$ they coalesce;
momentum + restitution: two equations

- Newton's experimental law: separation speed = e $\times$ approach speed
- Match each part of the diagram to the text.

Exercises 2.1·2.2

2.1 A 3 kg particle at 4 $\mathrm{m\,s^{-1}}$ catches a 1 kg particle at 1 $\mathrm{m\,s^{-1}}$, both moving right; they coalesce. Find the common speed and the energy lost. [4]

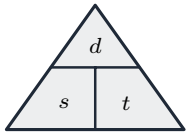


2.2 Two equal masses collide with $e = \frac{1}{2}$: A at 6 $\mathrm{m\,s^{-1}}$ hits B at rest. Find both speeds after. [3]



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2.1 $3(4)+1(1)=4v$: $v=3.25\ \mathrm{m\,s^{-1}}$. KE before $=24+0.5=24.5$ J; after $=\frac{1}{2}(4)(3.25)^2=21.125$ J; lost $=3.375$ J.

2.2 Two equal masses collide with $e=\frac{1}{2}$: A at 6 $\mathrm{m\,s^{-1}}$ hits B at rest. Find both speeds after. [3]



2.2 $6=v_A+v_B$ and $\frac{1}{2}(6)=v_B-v_A$: $v_B=4.5$, $v_A=1.5\ \mathrm{m\,s^{-1}}$ (A still moves forward, slower).

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Exercise 2.3

2.3 Using the loss formula loss $= \frac{m_A m_B}{2(m_A+m_B)}(1-e^2)(u_A-u_B)^2$, show that a direct collision conserves total kinetic energy only if $e = 1$ or the particles already have equal velocities. [2]



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2.3 The loss is zero exactly when $(1-e^2)(u_A-u_B)^2 = 0$: either $e = 1$ (perfectly elastic) or the particles were already moving together ($u_A = u_B$, no collision).

Exercise 3.1

3.1 *(Edexcel IAL WME02 June 2025 Q5a–c, condensed.)* Two particles P (mass $2m$) and Q (mass m) lie at rest on a smooth horizontal plane. P is projected with speed u directly towards Q, and they collide directly with coefficient of restitution e ($e > 0$).

(a) Show that the speed of Q immediately after the collision is $\frac{2(1+e)u}{3}$. (b) Show that the total kinetic energy lost in the collision is $k(1-e^2)mu^2$, where k is a constant to be determined. (c) Given that the magnitude of the impulse exerted on Q by P is $\frac{8}{9}mu$, find e. [12]



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3.1 (a) Momentum: $2mu = 2mv_P + mv_Q$; restitution: $eu = v_Q - v_P$. Substituting $v_P = v_Q - eu$: $2u = 2v_Q - 2eu + v_Q = 3v_Q - 2eu$, so $v_Q = \frac{2u(1+e)}{3}$ ✓. (b) From (a), $v_P = \frac{2(1+e)u}{3} - eu = \frac{(2-e)u}{3}$. KE after $= \frac{1}{2}(2m)\left(\frac{(2-e)u}{3}\right)^2 + \frac{1}{2}m\left(\frac{2(1+e)u}{3}\right)^2 = \frac{mu^2}{9}\left[(2-e)^2 + 2(1+e)^2\right] = \frac{mu^2}{9}(6+3e^2) = \frac{mu^2}{3}(2+e^2)$. Loss $= \frac{1}{2}(2m)u^2 - \frac{mu^2}{3}(2+e^2) = mu^2 - \frac{2mu^2}{3} - \frac{e^2mu^2}{3} = \frac{1}{3}(1-e^2)mu^2$, so $k = \frac{1}{3}$ ✓. (c) Impulse on Q $= m\Delta v$: $m \cdot \frac{2(1+e)u}{3} = \frac{8mu}{9} \Rightarrow \frac{2(1+e)}{3} = \frac{8}{9} \Rightarrow 1 + e = \frac{4}{3}$, so $e = \frac{1}{3}$.

Exercise 3.2

3.2 *(Original.)* A 2 kg particle moving right at 5 $\mathrm{m\,s^{-1}}$ collides directly with a 3 kg particle moving left at 1 $\mathrm{m\,s^{-1}}$; the coefficient of restitution is $\frac{1}{4}$. Find both velocities after the collision, and the total energy lost. [6]



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3.2 Right positive: momentum $2(5)+3(-1)=7=2v_A+3v_B$; restitution $e=\frac{v_B-v_A}{5-(-1)}=\frac{v_B-v_A}{6}=\frac{1}{4}$, so $v_B=v_A+1.5$. Then $2v_A+3v_A+4.5=7$: $v_A=0.5$, $v_B=2$ $\mathrm{m\,s^{-1}}$ (both right). KE before $=25+1.5=26.5$ J; after $=0.25+6=6.25$ J; lost $=20.25$ J.

Local review guidance: both equations with consistent signs; both speeds fully simplified before any energy statement; loss factorised to show the $(1-e^2)$ factor. This is authored classroom guidance.

Exercise 3.3 (a)

3.3 Two particles A, of mass 3 kg, and B, of mass 1 kg, move towards each other on a smooth horizontal surface. A has speed 4 $\mathrm{m\,s^{-1}}$ and B has speed 2 $\mathrm{m\,s^{-1}}$. After the collision A moves in its original direction with speed 2 $\mathrm{m\,s^{-1}}$.

(a) Find the velocity of B after the collision. [3]

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(a) Find the velocity of B after the collision. [3]

(a)

- momentum is conserved, so B moves at 4 $\mathrm{m\,s^{-1}}$ in A's original direction:

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B: \quad 3(4) + 1(-2) = 3(2) + 1v_B \Rightarrow v_B = \mathbf{4}\ \mathrm{m\,s^{-1}}$$

Exercise 3.3 (b)

3.3 Two particles A, of mass 3 kg, and B, of mass 1 kg, move towards each other on a smooth horizontal surface. A has speed 4 $\mathrm{m\,s^{-1}}$ and B has speed 2 $\mathrm{m\,s^{-1}}$. After the collision A moves in its original direction with speed 2 $\mathrm{m\,s^{-1}}$.

(b) Find the coefficient of restitution between A and B. [3]

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(b) Find the coefficient of restitution between A and B. [3]

(b)

- Newton's law of restitution, $e = \dfrac{\text{speed of separation}}{\text{speed of approach}}$:

$$e = \frac{v_B - v_A}{u_A - u_B} = \frac{4-2}{4-(-2)} = \frac{2}{6} = \mathbf{\frac{1}{3}}$$

Exercise 3.3 (c)

3.3 Two particles A, of mass 3 kg, and B, of mass 1 kg, move towards each other on a smooth horizontal surface. A has speed 4 $\mathrm{m\,s^{-1}}$ and B has speed 2 $\mathrm{m\,s^{-1}}$. After the collision A moves in its original direction with speed 2 $\mathrm{m\,s^{-1}}$.

(c) Find the kinetic energy lost in the collision. [3]

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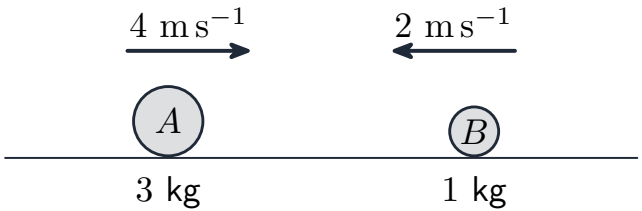
- the loss is the kinetic energy before minus after, $\frac{1}{2}mv^2$ for each particle:

$$\Delta E = \left[\tfrac{1}{2}(3)(4^2) + \tfrac{1}{2}(1)(2^2)\right] - \left[\tfrac{1}{2}(3)(2^2) + \tfrac{1}{2}(1)(4^2)\right] = 26 - 14 = \mathbf{12}\ \mathrm{J}$$

Exercise 3.3 (d)

3.3 Two particles A, of mass 3 kg, and B, of mass 1 kg, move towards each other on a smooth horizontal surface. A has speed 4 $\mathrm{m\,s^{-1}}$ and B has speed 2 $\mathrm{m\,s^{-1}}$. After the collision A moves in its original direction with speed 2 $\mathrm{m\,s^{-1}}$.

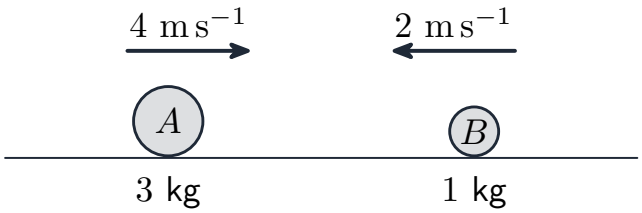
(d) Find the magnitude and direction of the impulse exerted on A by B. [2]



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(d) Find the magnitude and direction of the impulse exerted on A by B. [2]



(d)

- the impulse on A is its change in momentum, $I = m_A(v_A - u_A)$:

$$I = m_A(v_A - u_A) = 3(2 - 4) = -6 \text{ N s} \Rightarrow \mathbf{6} \text{ N s, opposite to A's original motion}$$