Exercise walk-through

7.3
Work, energy and power

Pearson Edexcel · International A-Level · Mathematics

Work, energy and power

- The **work-energy principle** : the **work done** by all the forces other than gravity equals the change in **kinetic energy** plus **potential energy**.
- **Conservation of mechanical energy** : if no forces other than gravity do work, KE + PE stays the same.
- **Power** is the rate of doing work. For a vehicle with driving force F moving at speed v, $P = Fv$. At a steady speed, the driving force equals the total of the forces against the motion.

Work, energy and power (3)

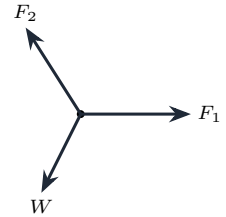
- The engine is then switched off, at 10 m s^{-1}. The car moves a distance d up the road before it stops: $\frac{1}{2} \times 750 \times 10^2 = 750g \times d\sin\alpha + 200d$, so $37\,500 = 1250d$ and $d = 30$ m. The resistance acts over the distance d along the road, and gravity over the height $d\sin\alpha$.
- Energy per unit mass: $\frac{1}{2}(20^2) = \frac{1}{2}(V^2 + 4V^2) + 10 \times 11$, so $200 = 2.5V^2 + 110$ and $V = 6$. The horizontal component, 12, never changes.

Collisions and restitution (2)

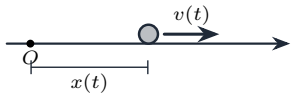
- Solving: $v_P = \dfrac{(2-e)u}{3}$ and $v_Q = \dfrac{2(1+e)u}{3}$.
- The **kinetic energy** after is $\frac{1}{2}(2m)v_P^2 + \frac{1}{2}mv_Q^2 = \dfrac{mu^2}{3}(2+e^2)$. Before, it was mu^2, so the loss is $\dfrac{mu^2}{3}(1-e^2)$.
- If the impulse on Q is $\frac{8}{9}mu$, then $m \times \dfrac{2(1+e)u}{3} = \dfrac{8mu}{9}$, so $e = \frac{1}{3}$, $v_Q = \frac{8}{9}u$ and $v_P = \frac{5}{9}u$.

Exercises 2.1·2.2

2.1 A crate of mass 20 kg is pushed 5 m up a slope at 30° to the horizontal against a frictional force of 15 N. Find the total work done against gravity and friction. [3]

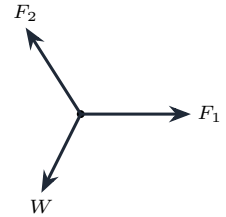


2.2 An engine works at 8 kW pulling a 1200 kg car along a level road with total resistance 500 N. Find the maximum constant speed and the acceleration at 8 $\mathrm{m\,s^{-1}}$. [4]



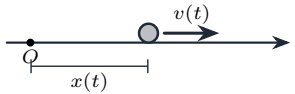
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2.1 Gravity: $20(9.8)(5\sin 30°) = 490$ J; friction: $15(5) = 75$ J; total 565 J.

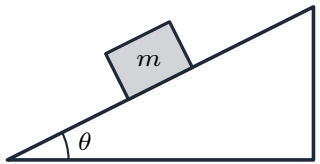
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2.2 Maximum speed: $\frac{8000}{500} = 16\ \mathrm{m\,s^{-1}}$. At 8: driving force = 1000 N, so $1000 - 500 = 1200a$ and $a = 0.417\ \mathrm{m\,s^{-2}}$ (0.42).

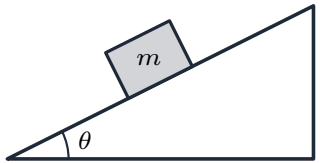
Exercise 2.3

2.3 A ball is projected at 15 $\mathrm{m\,s^{-1}}$ and reaches its highest point. Using energy ($g = 9.8$), find the height risen. [2]



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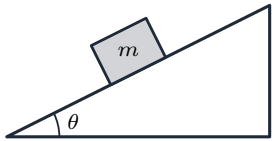
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2.3 $\frac{1}{2}(15)^2 = 9.8h$: $h = \frac{112.5}{9.8} = 11.5$ m.

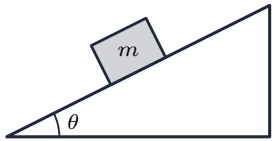
Exercise 3.1

3.1 *(Edexcel IAL WME02 June 2025 Q4.)* A rough plane is inclined at α to the horizontal, where $\sin\alpha = \frac{1}{7}$. A car of mass 750 kg moves up a line of greatest slope of the plane. The total non-gravitational resistance to the car's motion is 200 N and the engine works at a constant rate of 20 kW. (a) Find the constant speed at which the car can move up the plane. (b) Find the acceleration of the car at an instant when it is moving up the plane at 10 $\mathrm{m\,s^{-1}}$. (c) At the instant the car is moving at 10 $\mathrm{m\,s^{-1}}$ at the point A, the engine is switched off and the resistance remains 200 N. The car comes to rest at the point B. Use the work-energy principle to find the distance AB. [12]



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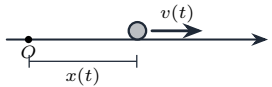
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3.1 (a) At constant speed the driving force $F = \frac{P}{V}$ balances resistance plus gravity: $\frac{20000}{V} = 750(9.8)\frac{1}{7} + 200 = 1050 + 200 = 1250$, so $V = 16$ $\mathrm{m\,s^{-1}}$. (b) $\frac{20000}{10} - 1250 = 750a$: $2000 - 1250 = 750a$, $a = 1$ $\mathrm{m\,s^{-2}}$. (c) Work-energy: KE lost = work against gravity + friction: $\frac{1}{2}(750)(10^2) = 750(9.8)(d\frac{1}{7}) + 200d$, i.e. $37500 = 1050d + 200d = 1250d$, so $AB = 30$ m.

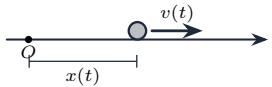
Exercise 3.2

3.2 *(Edexcel IAL WME02 June 2025 Q7.)* A small stone is projected from a point O on horizontal ground with speed 20 $\mathrm{m\,s^{-1}}$ at an angle α above the horizontal. The stone is modelled as a particle moving freely under gravity with $g = 10\ \mathrm{m\,s^{-2}}$. When the stone is 11 m above the ground, the magnitude of the vertical component of its velocity is $V\ \mathrm{m\,s^{-1}}$ and the magnitude of the horizontal component is $2V\ \mathrm{m\,s^{-1}}$. (a) Using conservation of mechanical energy, show that $V = 6$. (b) The stone hits a fixed plane, inclined at 60° to the horizontal, at the point A, the first point of contact, perpendicularly to the plane. Find the height of A above the ground (2 s.f.). [10]



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3.2 (a) Energy from O to the 11 m point: $\frac{1}{2}m(20)^2 = \frac{1}{2}m(V^2 + (2V)^2) + m(10)(11)$: $200 = \frac{5}{2}V^2 + 110$, so $V^2 = 36$ and $V = 6$ ✓ (components 6 and 12, speed$^2 = 180$). (b) Perpendicular to the 60° plane, the velocity makes 30° with the horizontal (below it), so with horizontal component 12 the speed is $\frac{12}{\cos 30°} = 8\sqrt{3}$ and speed$^2 = 192$. Energy from O to A: $200 = \frac{1}{2}(192) + 10H = 96 + 10H$, so $H = 10.4$, i.e. 10 m (2 s.f.).

Local review guidance: $\frac{P}{V}$ for the driving force with all three slope terms present; work-energy with resistance over the slope distance and gravity over the height; perpendicular impact converted to a velocity direction before any energy statement. This is authored classroom guidance.

Exercise 3.3 (a)

3.3 A car of mass 1000 kg drives up a straight road inclined at α to the horizontal, where $\sin\alpha = \frac{1}{20}$. The resistance to motion from non-gravitational forces is constant at 600 N. The engine works at a constant rate of 30 kW. Take $g = 9.8\ \mathrm{m\,s^{-2}}$.

(a) Find the maximum speed of the car up the road. [4]

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(a) Find the maximum speed of the car up the road. [4]

(a)

- at maximum speed the acceleration is zero, so the driving force balances resistance and the weight component: $F = 600 + 1000g\sin\alpha$

$$F = 600 + (1000)(9.8)\left(\tfrac{1}{20}\right) = 600 + 490 = 1090\ \text{N}$$

- power $P = Fv$:

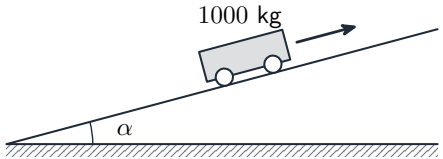
$$v = \frac{P}{F} = \frac{30\,000\ \text{W}}{1090\ \text{N}}$$

- $v_{\max} = \mathbf{27.5}\ \text{m s}^{-1}$

Exercise 3.3 (b)

3.3 A car of mass 1000 kg drives up a straight road inclined at α to the horizontal, where $\sin\alpha = \frac{1}{20}$. The resistance to motion from non-gravitational forces is constant at 600 N. The engine works at a constant rate of 30 kW. Take $g = 9.8\ \mathrm{m\,s^{-2}}$.

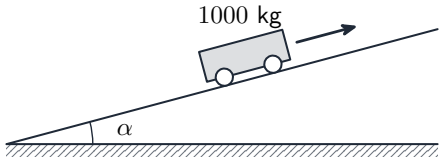
(b) Find the acceleration of the car at the instant its speed is $15\ \mathrm{m\,s^{-1}}$. [3]



Exercise 3.3 (b)

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(b) Find the acceleration of the car at the instant its speed is $15\ \text{m s}^{-1}$. [3]



(b)

- driving force at $15\ \text{m s}^{-1}$, $F = \dfrac{P}{v}$:

$$F = \frac{30\,000}{15} = 2000\ \text{N}$$

- Newton's second law up the slope, $F - 600 - 1000g\sin\alpha = 1000a$:

$$2000 - 1090 = 1000a$$

- $a = \mathbf{0.91}\ \text{m s}^{-2}$