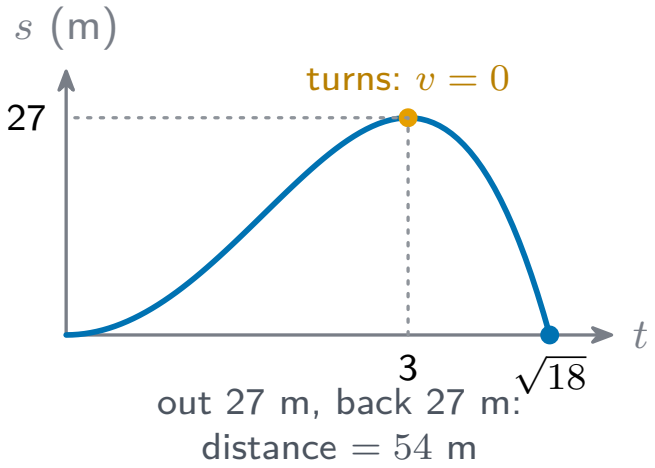
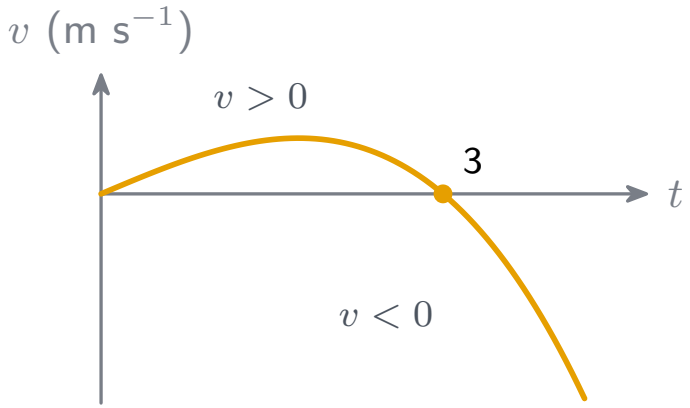
Exercise walk-through

7.2
Centres of mass

Pearson Edexcel · International A-Level · Mathematics

Figure: Kinematics with variable acceleration





- The particle stops at t = 3, 27 m from O, then comes back: the total distance is 27 + 27 = 54 m, while the displacement at return is 0
- Match each part of the diagram to the text.

Centres of mass: particles, laminas and wire frameworks

- For particles of masses $m_1, m_2, \ldots$ at $x_1, x_2, \ldots$, the **centre of mass** is at
- and in the same way for $\bar{y}$.
- For a uniform composite body, use a table:
- 1. split the body into pieces whose centres of mass you know (the booklet gives arcs, sectors, semicircles and triangles); 2. use area as the "mass" of a lamina, and length as the "mass" of a wire; 3. take moments about a convenient line.

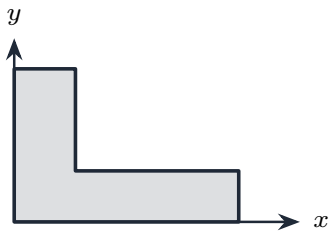
$$\bar{x} = \frac{\sum mx}{\sum m}$$

Work, energy and power (3)

- The engine is then switched off, at 10 m s^{-1}. The car moves a distance d up the road before it stops: $\frac{1}{2} \times 750 \times 10^2 = 750g \times d\sin\alpha + 200d$, so $37\,500 = 1250d$ and $d = 30$ m. The resistance acts over the distance d along the road, and gravity over the height $d\sin\alpha$.
- Energy per unit mass: $\frac{1}{2}(20^2) = \frac{1}{2}(V^2 + 4V^2) + 10 \times 11$, so $200 = 2.5V^2 + 110$ and $V = 6$. The horizontal component, 12, never changes.

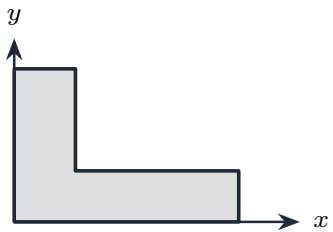
Exercise 2.1

2.1 Particles of mass 2 kg, 3 kg and 5 kg sit at $x = 0$, 4 and 6 on the x-axis. Find their centre of mass. [2]



Exercise 2.1

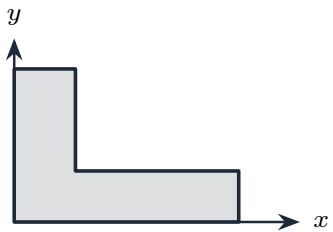
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2.1 $\bar{x} = \frac{2(0)+3(4)+5(6)}{10} = \frac{42}{10} = 4.2$ from the 2 kg particle.

Exercise 2.2

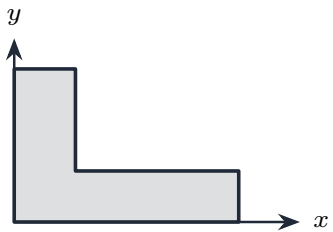
2.2 A uniform rectangular lamina 6 m by 4 m has a 2 m by 2 m square hole cut from one corner. Find the distance of the centre of mass from each outer edge adjacent to that corner. [4]



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Exercise 2.2

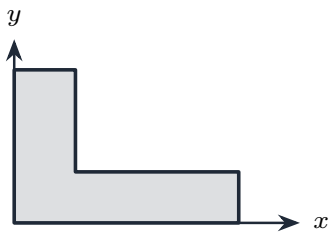
2.2 A uniform rectangular lamina 6 m by 4 m has a 2 m by 2 m square hole cut from one corner. Find the distance of the centre of mass from each outer edge adjacent to that corner. [4]



2.2 Rectangle (area 24) at (3, 2); hole (area -4) at (1, 1) from the adjacent corner: $\bar{x} = \frac{24(3)-4(1)}{20} = \frac{68}{20} = 3.4$ m and $\bar{y} = \frac{24(2)-4(1)}{20} = \frac{44}{20} = 2.2$ m from the two outer edges at that corner.

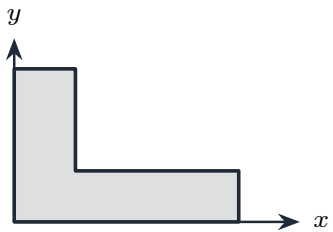
Exercise 2.3

2.3 Three uniform rods form a uniform wire triangle with sides 5, 5 and 6. Find the distance of the centre of mass from the 6-side. [3]



Exercise 2.3

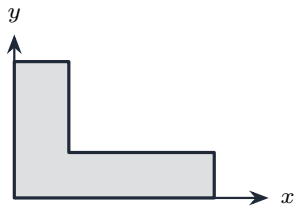
2.3 Three uniform rods form a uniform wire triangle with sides 5, 5 and 6. Find the distance of the centre of mass from the 6-side. [3]



2.3 The two 5-sides each have midpoint 1.5 m above the 6-side (height of the 5-5-6 triangle $= 4$); the 6-side has COM distance 0. Weights by length: $\bar{y} = \frac{5(1.5)+5(1.5)+6(0)}{16} = \frac{15}{16} = 0.9375$ m from the 6-side.

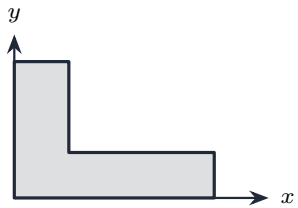
Exercise 3.1

3.1 *(Edexcel IAL WME02 June 2025 Q3, condensed.)* A uniform rigid wire framework consists of two semicircular arcs AB and CD, both of centre O, joined by two straight pieces AC and DB. The arc AB has radius $2a$, the arc CD has radius a, $AC = DB = a$, and ACDB is a straight line. The arcs lie on opposite sides of ACDB. The centre of mass of a semicircular arc of radius r is $\frac{2r}{\pi}$ from its centre. (a) Show that the centre of mass of the framework is $\frac{6a}{3\pi+2}$ from AB. (b) The point E lies on the arc AB with $\angle AOE = 90°$. The framework is freely suspended from A and hangs in equilibrium with AE at angle θ to the downward vertical. Show that $\tan\theta = \dfrac{3\pi - 1}{3\pi + 5}$. [10]



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Full solution on next slide

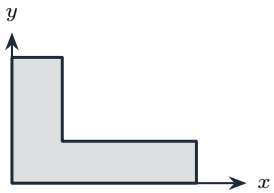
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Full solution 3.1

3.1 (a) Table about the line AB: arc AB length $2\pi a$, COM $\frac{4a}{\pi}$ above; arc CD length πa, COM $\frac{2a}{\pi}$ BELOW; straights total $2a$ on the line: $2\pi a\cdot\frac{4a}{\pi}-\pi a\cdot\frac{2a}{\pi}+2a\cdot 0=8a^2-2a^2=6a^2$; total length $=(3\pi+2)a$; so $\bar{x}=\frac{6a^2}{(3\pi+2)a}=\frac{6a}{3\pi+2}$ ✓ (the COM is on the AB side, distance d from the line). (b) By symmetry G lies on OE. In triangle AOE, $OA=OE=2a$ and the angle at O is 90°, so $\angle OAE=45°$; the COM is at distance d from A's vertical: in right triangle AOG, $\tan\angle OAG=\frac{d}{2a}=\frac{6}{2(3\pi+2)}=\frac{3}{3\pi+2}$. Hanging from A puts G on the vertical, so $\theta=\angle GAE=\angle OAE-\angle OAG=45°-\angle OAG$, and $\tan\theta=\dfrac{1-\frac{3}{3\pi+2}}{1+\frac{3}{3\pi+2}}=\dfrac{3\pi-1}{3\pi+5}$ ✓.

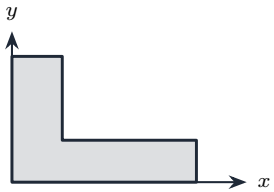
Exercise 3.2

3.2 *(Original.)* A uniform semicircular lamina of radius 6 cm is freely suspended from one end P of its straight edge. Find the angle between the straight edge and the vertical, giving an exact tangent. [4]



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3.2 The COM sits on the symmetry axis at $\frac{4r}{3\pi} = \frac{8}{\pi}$ cm from the centre O, i.e. $\frac{8}{\pi}$ from the straight edge, on the perpendicular through O. Suspended from P (one end of the edge, $OP = 6$), G hangs below P. In the right triangle with legs $OP = 6$ (along the edge) and the COM's perpendicular distance $\frac{8}{\pi}$ from the edge: the angle θ between the edge and the vertical satisfies $\tan\theta = \dfrac{8/\pi}{6} = \dfrac{4}{3\pi}$.

Local review guidance: pieces weighted by area/length with the correct booklet distance; opposite-side pieces signed; suspension angles via right-triangle trigonometry at the suspension point plus isosceles geometry. This is authored classroom guidance.

Exercise 3.3 (a)

3.3 The uniform lamina $OABCDE$ is L-shaped, as shown. $OA = OE = 8$ cm, $AB = DE = 2$ cm, and all the corners are right angles.

(a) Find the distance of the centre of mass of the lamina from OA and from OE. [5]

Exercise 3.3 (a)

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(a) Find the distance of the centre of mass of the lamina from OA and from OE. [5]

(a)

- split into a vertical rectangle 8×2 (area 16, centre $(1, 4)$) and a horizontal rectangle 6×2 (area 12, centre $(5, 1)$), with x measured from OA and y from OE
- $\bar{x} = \dfrac{\sum A_i x_i}{\sum A_i}$ and $\bar{y} = \dfrac{\sum A_i y_i}{\sum A_i}$:

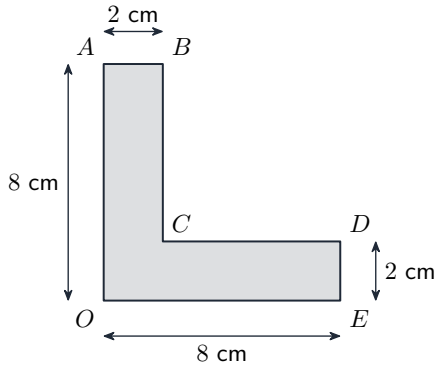
$$\bar{x} = \frac{16(1) + 12(5)}{28} = \frac{19}{7}, \qquad \bar{y} = \frac{16(4) + 12(1)}{28} = \frac{19}{7}$$

- the centre of mass is $\mathbf{\frac{19}{7}}$ cm (2.71 cm) from both OA and OE

Exercise 3.3 (b)

3.3 The uniform lamina $OABCDE$ is L-shaped, as shown. $OA = OE = 8$ cm, $AB = DE = 2$ cm, and all the corners are right angles.

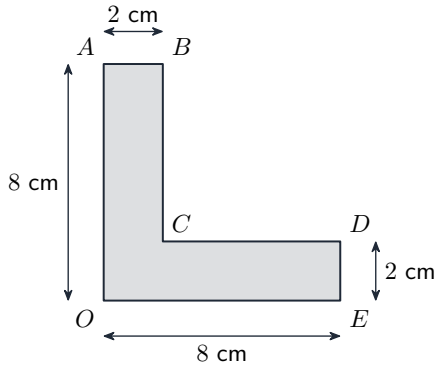
(b) The lamina is freely suspended from A and hangs in equilibrium. Find the angle between OA and the vertical. [3]



Exercise 3.3 (b)

3.3 The uniform lamina $OABCDE$ is L-shaped, as shown. $OA = OE = 8$ cm, $AB = DE = 2$ cm, and all the corners are right angles.

(b) The lamina is freely suspended from A and hangs in equilibrium. Find the angle between OA and the vertical. [3]



(b)

- hanging from A, AG is vertical; G is $\frac{19}{7}$ cm from OA and $8 - \frac{19}{7} = \frac{37}{7}$ cm below A along OA:

$$\tan\theta = \frac{19/7}{37/7} = \frac{19}{37} \Rightarrow \theta = \mathbf{27.2}^\circ$$