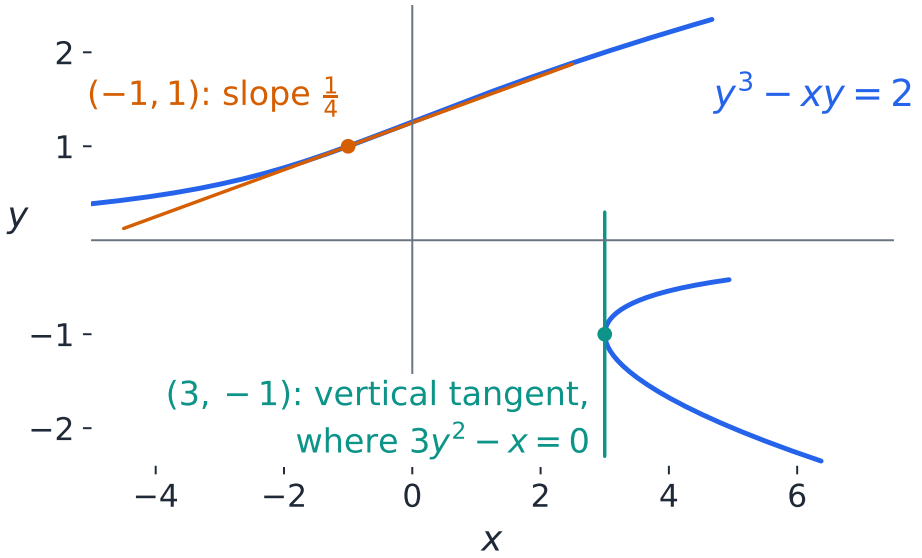
Exercise walk-through

4.5
Integration methods and volumes of revolution

Pearson Edexcel · International A-Level · Mathematics

Figure: Implicit and parametric differentiation



- Where the denominator of $\frac{dy}{dx}$ is zero (and the numerator is not), the tangent is vertical
- Read the axes and units first.

Integration: substitution, parts and partial fractions

- **Substitution** (usually given): rewrite the integral in u, adjust dx via $\frac{du}{dx}$, integrate, substitute back. It is the chain rule reversed.
- **Integration by parts** : $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$ — the product rule reversed. Know $\int \ln x\,dx = x\ln x - x + c$ and repeated parts for $\int x^2 e^x dx$-type integrals.

$$\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$$

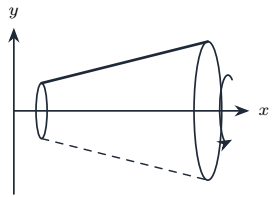
Integration: substitution, parts and partial fractions (2)

- **Partial fractions** turn a **rational** integrand into sums of $\frac{A}{ax+b}$ (integrating to logs) and $\frac{C}{(ax+b)^2}$ (to powers).
- Parts with $u = \mathrm{e}^x$, $\frac{dv}{dx} = \cos x$:
- Parts again on the new integral ($u = \mathrm{e}^x$, $\frac{dv}{dx} = \sin x$):

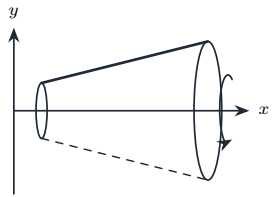
$$= \mathrm{e}^x \sin x + \mathrm{e}^x \cos x - \int \mathrm{e}^x \cos x \, dx$$

Exercises 2.1·2.2

2.1 Use the substitution $u = x^2 + 1$ to find $\int x(x^2+1)^3\,dx$. [3]

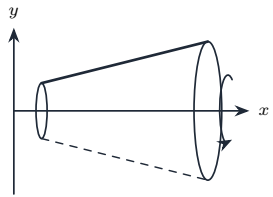


2.2 Find $\int xe^{2x}\,dx$ by parts. [3]



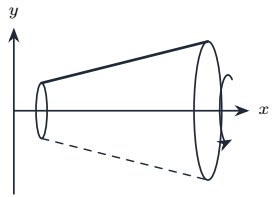
Exercises 2.1·2.2

2.1 Use the substitution $u = x^2 + 1$ to find $\int x(x^2+1)^3\,dx$. [3]



2.1 $du = 2x\,dx$: $\frac{1}{2}\int u^3 du = \frac{(x^2+1)^4}{8} + c$.

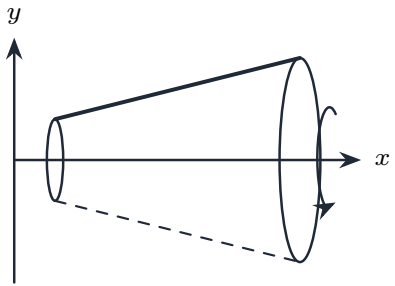
2.2 Find $\int x\mathrm{e}^{2x}\,dx$ by parts. [3]



2.2 $\frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{4}\mathrm{e}^{2x} + c$ (parts with $u = x$, $\frac{dv}{dx} = \mathrm{e}^{2x}$).

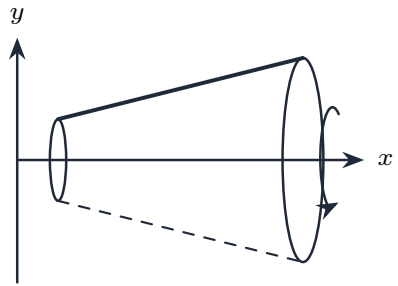
Exercise 2.3

2.3 Find $\int \ln 3x \, dx$. [3]



Exercise 2.3

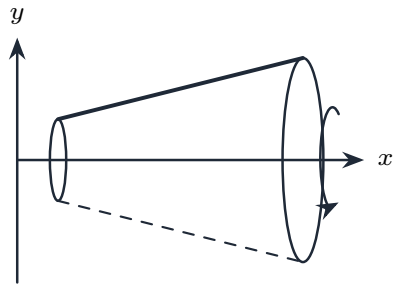
2.3 Find $\int \ln 3x \, dx$. [3]



2.3 $x \ln 3x - x + c$ (parts with $u = \ln 3x$, $\frac{dv}{dx} = 1$).

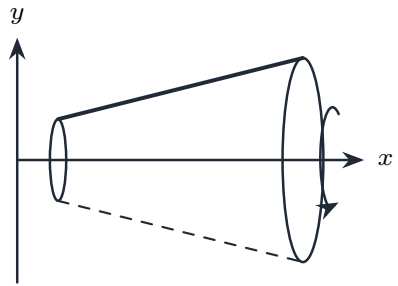
Exercise 2.4

2.4 Find $\int \frac{4x+5}{(x+1)(x+2)}\,dx$ using partial fractions. [5]



Exercise 2.4

2.4 Find $\displaystyle\int \frac{4x+5}{(x+1)(x+2)}\,dx$ using partial fractions. [5]

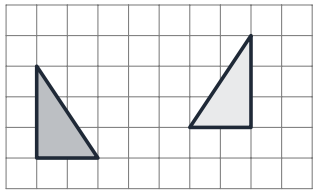


2.4 $\dfrac{4x+5}{(x+1)(x+2)} = \dfrac{1}{x+1} + \dfrac{3}{x+2}$ (check at $x=0$: LHS $\frac{5}{2}$, RHS $1+\frac{3}{2}$ ✓):

$$\ln|x+1| + 3\ln|x+2| + c$$

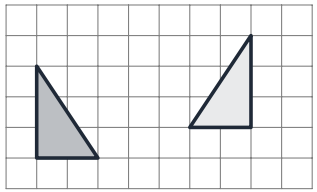
Exercise 2.5

2.5 Find the volume when $y = \sqrt{x}$ on $[0, 4]$ rotates about the x-axis. [4]



Exercise 2.5

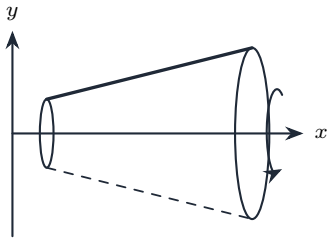
2.5 Find the volume when $y = \sqrt{x}$ on $[0, 4]$ rotates about the x-axis. [4]



2.5 $V = \pi \displaystyle\int_0^4 x \, dx = \pi \left[\frac{x^2}{2} \right]_0^4 = 8\pi.$

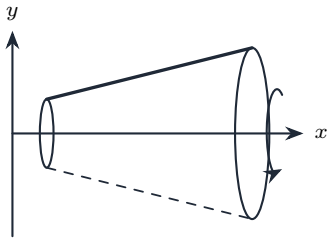
Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q7.)* Use the substitution $u = \tan x$ to find $\int \tan x \sec^2 x \, dx$. [4]



Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q7.)* Use the substitution $u = \tan x$ to find $\displaystyle\int \tan x \sec^2 x \, dx$. [4]



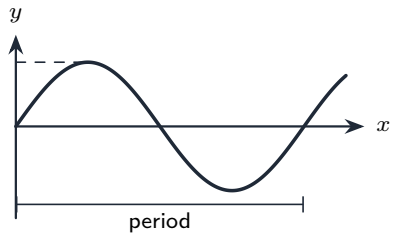
3.1 $u = \tan x$: $\dfrac{du}{dx} = \sec^2 x$, so $\sec^2 x \, dx = du$:

$$\int \tan x \sec^2 x \, dx = \int u \, du = \frac{u^2}{2} + c = \frac{\tan^2 x}{2} + c$$

- Check: $\dfrac{d}{dx}\left(\frac{1}{2}\tan^2 x\right) = \tan x \sec^2 x$ ✓.

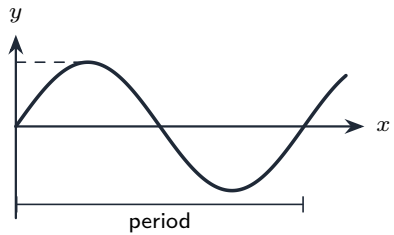
Exercise 3.2

3.2 *(Structure adapted from Edexcel IAL WMA14 June 2025 Q9.)* The region under $y = 2\sin x$ for $0 \leq x \leq \pi$ rotates about the x-axis. Find the exact volume, showing your use of a double-angle identity. [6]



Exercise 3.2

3.2 *(Structure adapted from Edexcel IAL WMA14 June 2025 Q9.)* The region under $y = 2\sin x$ for $0 \le x \le \pi$ rotates about the x-axis. Find the exact volume, showing your use of a double-angle identity. [6]



3.2 $V = \pi \displaystyle\int_0^{\pi} 4\sin^2 x\,dx = 4\pi \int_0^{\pi} \tfrac{1}{2}(1-\cos 2x)\,dx = 2\pi\left[x - \frac{\sin 2x}{2}\right]_0^{\pi} = 2\pi^2.$

- Check: the mean of $\sin^2$ over a full half-turn is $\frac{1}{2}$: $4\pi \times \frac{1}{2} \times \pi = 2\pi^2$ ✓ exactly.

Local review guidance: substitution applied to dx too, or parts with the right u; exact antiderivatives and volumes; identity used before integrating a squared trig function. This is authored classroom guidance.