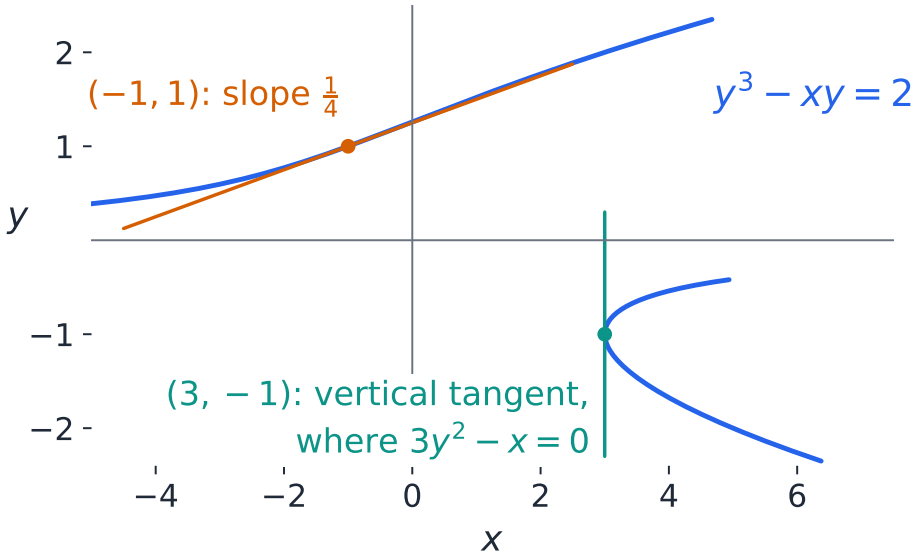
Exercise walk-through

4.1
Proof by contradiction and partial fractions

Pearson Edexcel · International A-Level · Mathematics

Figure: Implicit and parametric differentiation



- Where the denominator of $\frac{dy}{dx}$ is zero (and the numerator is not), the tangent is vertical
- Read the axes and units first.

Integration: substitution, parts and partial fractions

- **Substitution** (usually given): rewrite the integral in u, adjust dx via $\frac{du}{dx}$, integrate, substitute back. It is the chain rule reversed.
- **Integration by parts** : $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$ — the product rule reversed. Know $\int \ln x\,dx = x\ln x - x + c$ and repeated parts for $\int x^2 e^x dx$-type integrals.

$$\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$$

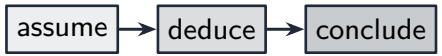
Integration: substitution, parts and partial fractions (2)

- **Partial fractions** turn a **rational** integrand into sums of $\frac{A}{ax+b}$ (integrating to logs) and $\frac{C}{(ax+b)^2}$ (to powers).
- Parts with $u = \mathrm{e}^x$, $\frac{dv}{dx} = \cos x$:
- Parts again on the new integral ($u = \mathrm{e}^x$, $\frac{dv}{dx} = \sin x$):

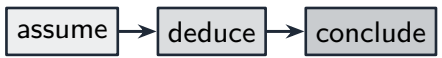
$$= \mathrm{e}^x \sin x + \mathrm{e}^x \cos x - \int \mathrm{e}^x \cos x \, dx$$

Exercises 2.1·2.2

2.1 Prove by contradiction that $\sqrt{3}$ is irrational. [4]



2.2 Decompose $\dfrac{7x+10}{(x+1)(x+3)}$. [3]



Exercises 2.1·2.2

2.1 Prove by contradiction that $\sqrt{3}$ is irrational. [4]

assume → deduce → conclude

2.1 Suppose $\sqrt{3} = \frac{p}{q}$ in lowest terms. Then $3q^2 = p^2$, so p^2 is divisible by 3, so p is (if $p = 3m$, $p^2 = 9m^2$; conversely a prime dividing p^2 divides p). Write $p = 3m$: $3q^2 = 9m^2 \Rightarrow q^2 = 3m^2$, so q is divisible by 3 too – both divisible contradicts lowest terms. ■

2.2 Decompose $\dfrac{7x + 10}{(x + 1)(x + 3)}$. [3]

assume → deduce → conclude

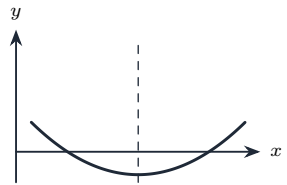
2.2 $7x + 10 = A(x + 3) + B(x + 1)$: at $x = -1$, $A = \frac{3}{2}$; at $x = -3$, $B = \frac{11}{2}$. (Or compare coefficients: $A + B = 7$, $3A + B = 10$.)

Exercises 2.3·2.4

2.3 Decompose $\dfrac{5x+7}{(x-1)^2}$. [3]

assume → deduce → conclude

2.4 Write $\dfrac{x^2+2x+5}{x^2+x}$ in the form $1+\dfrac{A}{x}+\dfrac{B}{x+1}$. [4]



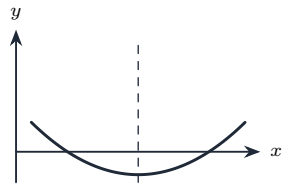
Exercises 2.3·2.4

2.3 Decompose $\dfrac{5x+7}{(x-1)^2}$. [3]

assume → deduce → conclude

2.3 With $\dfrac{5x+7}{(x-1)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2}$, multiply through: $5x+7 = A(x-1)+B$. Comparing x-coefficients, $A=5$; comparing constants, $7 = -5 + B \Rightarrow B = 12$.

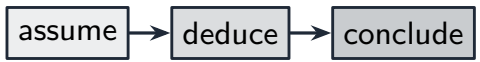
2.4 Write $\dfrac{x^2+2x+5}{x^2+x}$ in the form $1 + \dfrac{A}{x} + \dfrac{B}{x+1}$. [4]



2.4 $\dfrac{x^2+2x+5}{x^2+x} = 1 + \dfrac{x+5}{x^2+x} = 1 + \dfrac{x+5}{x(x+1)}$; decompose $\dfrac{x+5}{x(x+1)} = \dfrac{5}{x} - \dfrac{4}{x+1}$. So $A=5$, $B=-4$. Check at $x=1$: $1+5-2=4$; original $\frac{8}{2}=4$ ✓.

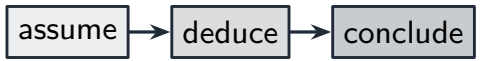
Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q10.)* Use proof by contradiction to show that $|\sin x + \cos x| < 1$ for all angles x with $90° < x < 180°$. [4]



Exercise 3.1

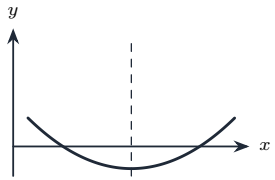
3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q10.)* Use proof by contradiction to show that $|\sin x + \cos x| < 1$ for all angles x with $90° < x < 180°$. [4]



3.1 Suppose some x in $90° < x < 180°$ has $|\sin x + \cos x| \geq 1$. Squaring: $1 + \sin 2x \geq 1$, so $\sin 2x \geq 0$. But $180° < 2x < 360°$ forces $\sin 2x < 0$ – a contradiction. So $|\sin x + \cos x| < 1$ throughout. ■ Check at $x = 135°$: $|0| = 0 < 1$ ✓.

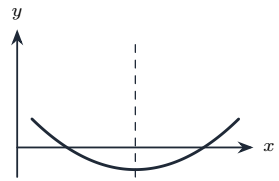
Exercise 3.2

3.2 Decompose $\dfrac{2x^2+5x+4}{(x+2)(x+1)^2}$ and verify your decomposition at $x=1$. [5]



Exercise 3.2

3.2 Decompose $\dfrac{2x^2+5x+4}{(x+2)(x+1)^2}$ and verify your decomposition at $x=1$. [5]



3.2 $\dfrac{2x^2+5x+4}{(x+2)(x+1)^2} = \dfrac{A}{x+2}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}$: $2x^2+5x+4 = A(x+1)^2+B(x+2)(x+1)+C(x+2)$.

- $x=-2$: $8-10+4=2=A$. $x=-1$: $2-5+4=1=C$. x^2: $2=A+B \Rightarrow B=0$.

$$\frac{2}{x+2}+\frac{1}{(x+1)^2}$$

Check at $x=1$: LHS $=\frac{2+5+4}{(1+2)(1+1)^2}=\frac{11}{12}$; RHS $=\frac{2}{3}+\frac{1}{4}=\frac{11}{12}$ ✓.

Local review guidance: assumption of the negation stated, or correct partial-fraction identity set up; exact values; verification or the named contradiction. This is authored classroom guidance.