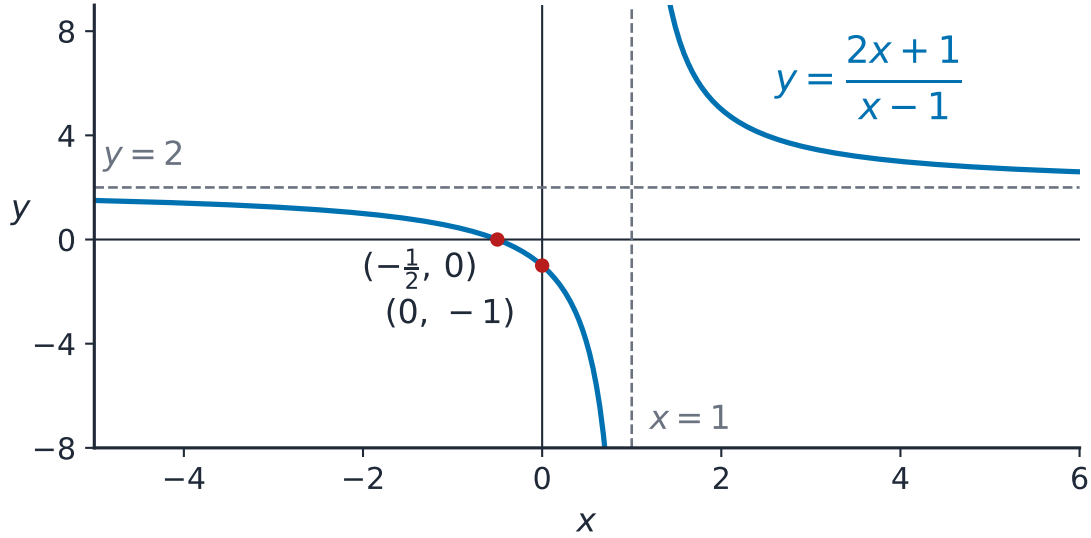
Exercise walk-through

3.4
Exponentials, logarithms and log-graph models

Pearson Edexcel · International A-Level · Mathematics

Figure: Algebra: rational expressions and functions



- Asymptotes first, then intercepts, then the two branches
- Read the axes and units first.

Exponentials, logarithms and log-graph models

- Gradient: $\dfrac{0-(-2)}{-3.5-0} = -\dfrac{4}{7}$; intercept -2:
- Convert: $y = 10^{-\frac{4}{7}\log_{10} x - 2} = 10^{-2}\, x^{-4/7}$, so
- Check with a point on the line: $\log_{10} x = 0 \Rightarrow x = 1$, $\log_{10} y = -2 \Rightarrow y = \frac{1}{100}$ ✓ and the formula gives $y = \frac{1}{100} \cdot 1 = \frac{1}{100}$ ✓.

$$\log_{10} y = -\tfrac{4}{7}\log_{10} x - 2$$

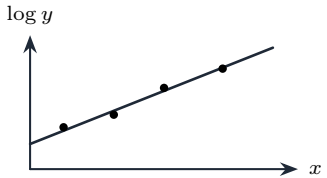
Numerical methods: sign changes and iteration

- **Sign change** : if f is continuous on $[a, b]$ and $f(a)f(b) < 0$, a root lies between a and b. Halve the interval to pin a root to any accuracy.
- **Iteration** : a rearrangement $x_{n+1} = f(x_n)$ from a start value converges to a **fixed point** (or not — always check against a sign-change bound).
- Check the **iteration** converges: $|x_3 - x_2|$ small and the values settle toward 0.43 ✓.
- g is continuous, and the **sign change** between 0.4295 and 0.4305 brackets a root, so the root is 0.430 to 3 d.p. ✓.

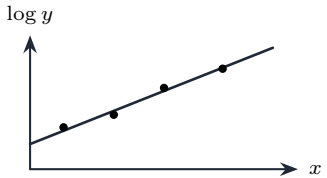
$$x_2 = \tfrac{1}{3}\arctan\left(\tfrac{3}{0.8}\right) = \tfrac{1}{3}\arctan(3.75) = 0.4367 \qquad x_4 = 0.4307 \text{ (4 d.p.)}$$

Exercises 2.1·2.2

2.1 Solve $e^{2x} = 50$, giving x to 3 significant figures. [2]

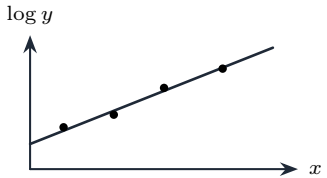


2.2 Solve $\ln(3x - 2) = 4$ exactly. [2]



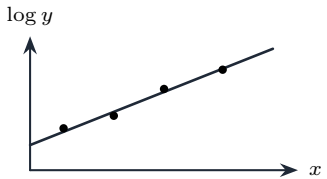
Exercises 2.1·2.2

2.1 Solve $e^{2x} = 50$, giving x to 3 significant figures. [2]



2.1 $x = \frac{1}{2}\ln 50 = 1.96$ (3 s.f.).

2.2 Solve $\ln(3x - 2) = 4$ exactly. [2]

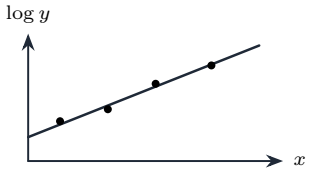


2.2 $3x - 2 = e^4$, so $x = \dfrac{e^4 + 2}{3}$.

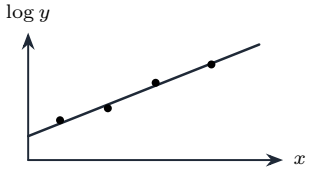
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Exercises 2.3·2.4

2.3 A population is modelled by $P = 2000\mathrm{e}^{0.05t}$. Find P when $t = 10$ and state the model's initial value. [2]

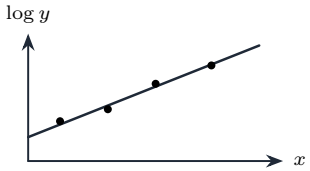


2.4 For $y = ax^n$, a $\log y$–$\log x$ plot has gradient 3 and intercept 0.778 ($= \log 6$). Find a and n. [2]



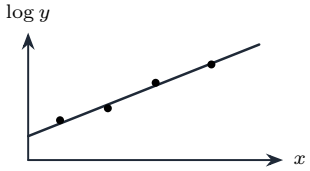
Exercises 2.3·2.4

2.3 A population is modelled by $P = 2000\mathrm{e}^{0.05t}$. Find P when $t = 10$ and state the model's initial value. [2]



2.3 $P = 2000\mathrm{e}^{0.5} = 3297$; initial value 2000 (at $t = 0$).

2.4 For $y = ax^n$, a $\log y$–$\log x$ plot has gradient 3 and intercept 0.778 ($= \log 6$). Find a and n. [2]

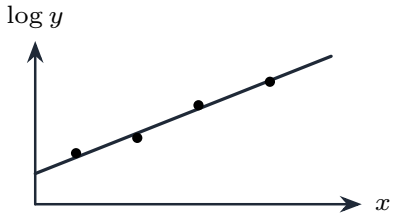


2.4 $n = 3$ and $\log a = \log 6 \Rightarrow a = 6$: $y = 6x^3$.

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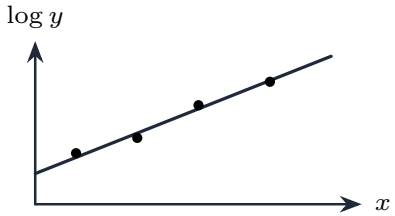
Exercise 2.5

2.5 Write $\log_{10} y = 2x + 1$ as an exponential relationship between y and x. [2]



Exercise 2.5

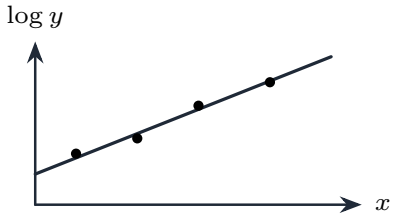
2.5 Write $\log_{10} y = 2x + 1$ as an exponential relationship between y and x. [2]



2.5 $y = 10^{2x+1} = 10 \times 100^{x}$.

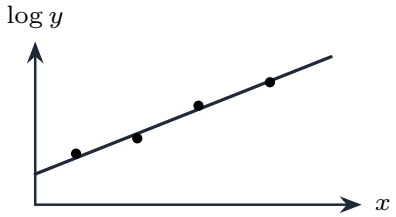
Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q3.)* A $\log_{10} y$ against $\log_{10} x$ line passes through $(-2, 1)$ and $(0, 5)$. Express y in the form px^q, giving p and q exactly. [5]



Exercise 3.1

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3.1 Gradient $= \dfrac{5-1}{0-(-2)} = 2$; intercept 5:

$$\log_{10} y = 2\log_{10} x + 5 \implies y = 10^5\, x^2 = 100\,000\, x^2$$

$p = 100\,000$, $q = 2$. Check at $(0, 5)$: $x = 1$, $y = 10^5$, $\log y = 5$ ✓.

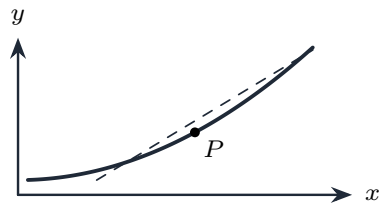
Exercise 3.2

3.2 *(Adapted from Edexcel IAL WMA13 June 2025 Q6.)* The temperature of an engine is modelled by $\theta = 25 + Ae^{-kt}$ (θ in °C, t in minutes). The engine is 95 °C when switched off at $t = 0$, and 40 °C at $t = 6$.

(a) Find A and k (3 s.f.).

(b) Find the time at which the temperature is decreasing at 4 °C per minute (2 d.p.).

(c) Explain why the model cannot hold below 25 °C. [8]



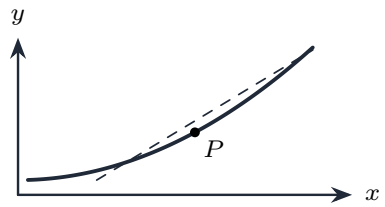
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Full solution on next slide

Full solution 3.2

3.2 (a) At $t = 0$: $95 = 25 + A \Rightarrow A = 70$. At $t = 6$: $40 = 25 + 70\mathrm{e}^{-6k} \Rightarrow \mathrm{e}^{-6k} = \frac{15}{70} = \frac{3}{14}$, so

$$k = \tfrac{1}{6} \ln \tfrac{14}{3} = 0.257 \text{ (3 s.f.)}$$

(b) $\dfrac{d\theta}{dt} = -70k\mathrm{e}^{-kt} = -4$:

$$\mathrm{e}^{-kt} = \frac{4}{70 \times 0.257} = 0.2226 \implies t = \frac{\ln(1/0.2226)}{0.257} = 5.85 \text{ (2 d.p.)}$$

Check: $\theta(5.85) = 25 + 70\mathrm{e}^{-0.257(5.85)} = 25 + 70(0.2226) = 40.6$ °C, falling at $70 \times 0.257 \times 0.2226 = 4.0$ °C/min ✓. (c) As $t \to \infty$, $\theta \to 25$ °C from above: the model's floor is room temperature, so it never predicts values below 25 °C – a colder-than-room prediction needs a new model.

Local review guidance: correct log recovery or correct differentiation of the model; exact/rounded values as asked; model interpretation tied to the limit. This is authored classroom guidance.

Exercise 3.3 (a)

3.3 The number of bacteria, N, in a culture t hours after it is set up is modelled by $N = A\mathrm{e}^{kt}$, where A and k are constants. The graph of $\ln N$ against t is a straight line through the points $(0,\ 4.2)$ and $(6,\ 6.0)$.

(a) Find the values of A and k, giving A to 3 significant figures. [4]

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(a) Find the values of A and k, giving A to 3 significant figures. [4]

(a)

- $\ln N = \ln A + kt$ is a straight line with gradient $k = \frac{6.0-4.2}{6-0} = \mathbf{0.3}$ and intercept $\ln A = 4.2$, so $A = \mathrm{e}^{4.2} = \mathbf{66.7}$

Exercise 3.3 (b)

3.3 The number of bacteria, N, in a culture t hours after it is set up is modelled by $N = A\mathrm{e}^{kt}$, where A and k are constants. The graph of $\ln N$ against t is a straight line through the points $(0,\ 4.2)$ and $(6,\ 6.0)$.

(b) Use the model to predict the number of bacteria after 10 hours. [2]

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(b) Use the model to predict the number of bacteria after 10 hours. [2]

(b)

- substitute $t = 10$: $N = A\mathrm{e}^{kt} = 66.69\,\mathrm{e}^{0.3(10)} = 66.69\,\mathrm{e}^{3} = \mathbf{1340}$ (3 s.f.)

Exercise 3.3 (c)

3.3 The number of bacteria, N, in a culture t hours after it is set up is modelled by $N = A\mathrm{e}^{kt}$, where A and k are constants. The graph of $\ln N$ against t is a straight line through the points $(0,\ 4.2)$ and $(6,\ 6.0)$.

(c) Find the time at which the number of bacteria first reaches 1000. [2]

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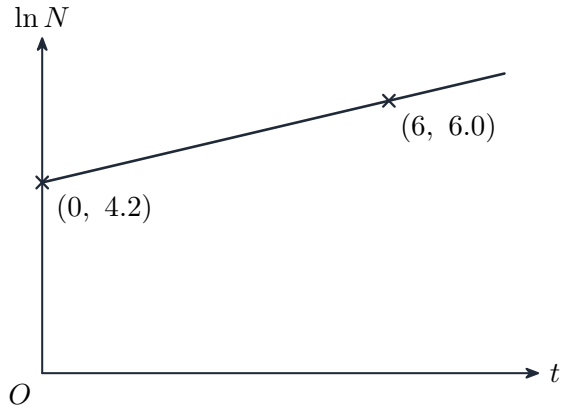
(c)

- set $N = 1000$ in $\ln N = 4.2 + 0.3t$: $0.3t = \ln 1000 - 4.2 = 2.7078$, so $t =$ **9.03** hours

Exercise 3.3 (d)

3.3 The number of bacteria, N, in a culture t hours after it is set up is modelled by $N = Ae^{kt}$, where A and k are constants. The graph of $\ln N$ against t is a straight line through the points $(0,\ 4.2)$ and $(6,\ 6.0)$.

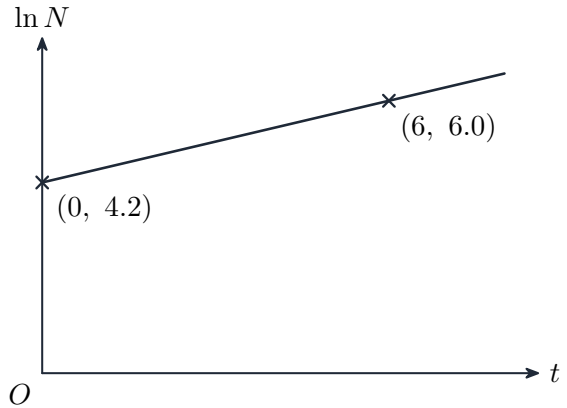
(d) Give one reason why the model may not be reliable for predicting N after 48 hours. [1]



Exercise 3.3 (d)

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(d)

- growth cannot go on for ever: food and space run out