Exercise walk-through

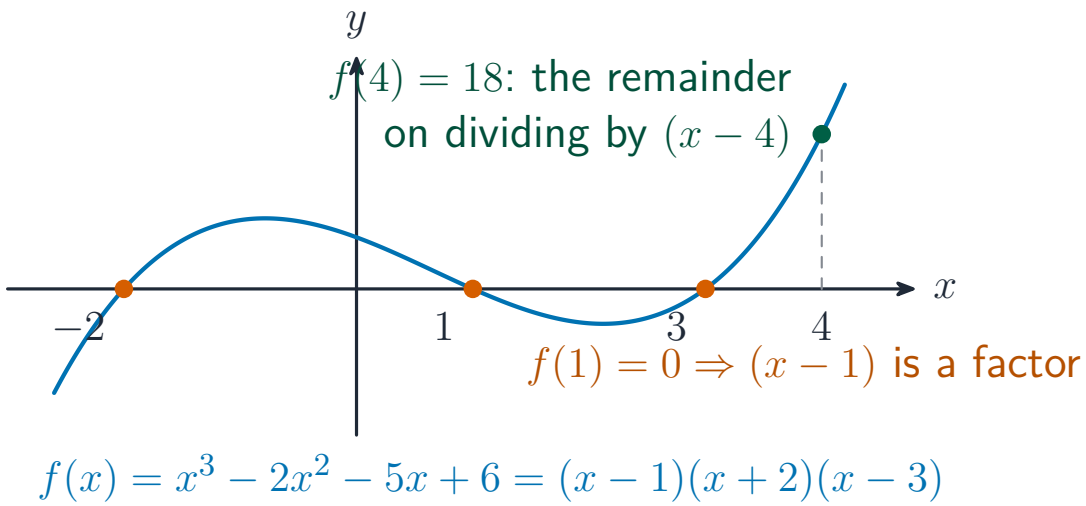
2.1
Proof: exhaustion and counterexample

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Proof

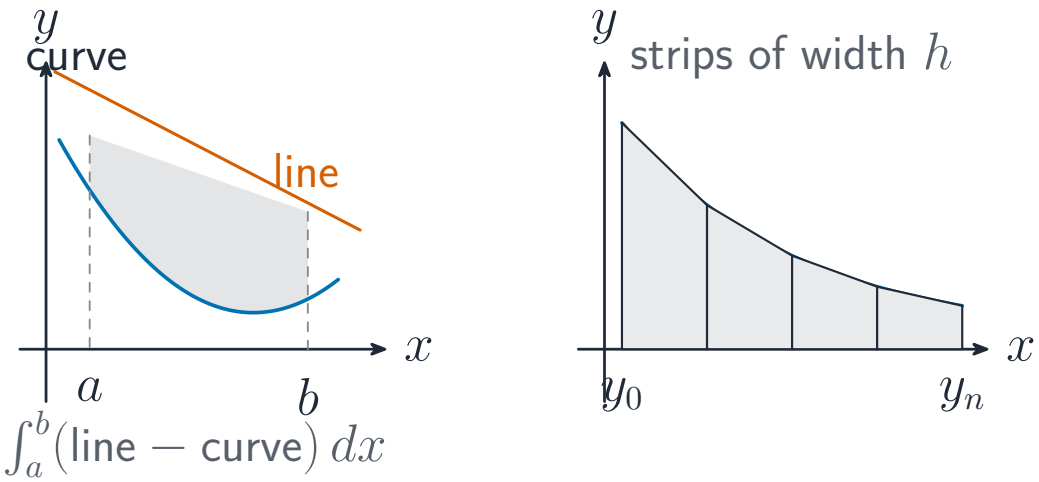
- **Proof by exhaustion** : split the possibilities into cases and check every case.
- **Disproof by counterexample** : one example that breaks the claim disproves it.
- Algebraic **proof**: start from what is given, manipulate identities (not equations to solve), and end with the target statement.

Figure: Algebra: the factor and remainder theorems



- Roots are factors; the height at x = b is the remainder on dividing by (x − b)
- Read the axes and units first.

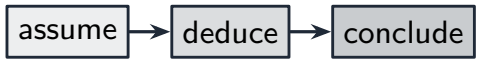
Figure: Integration: areas and the trapezium rule



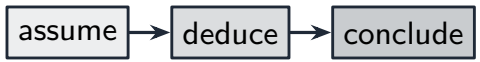
- Area between uses $\int(\text{upper} - \text{lower})$, splitting at crossings; the **trapezium rule** joins neighbouring values with straight strips.
- Read the axes and units first.

Exercises 2.1·2.2

2.1 Disprove by counterexample: "$n^2 + 1$ is prime for all positive integers n." [2]

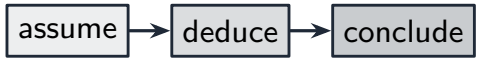


2.2 Prove that the sum of three consecutive integers is divisible by 3. [3]



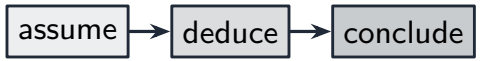
Exercises 2.1·2.2

2.1 Disprove by counterexample: "$n^2 + 1$ is prime for all positive integers n." [2]



2.1 $n = 3$: $9 + 1 = 10 = 2 \times 5$, which is not prime. One counterexample disproves the claim.

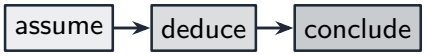
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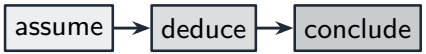
2.2 Let the integers be $(n-1)+n+(n+1) = 3n$, which is divisible by 3 for every integer n. ■

Exercises 2.3·2.4

2.3 Prove by exhaustion that $n^2 - 1$ is divisible by 3 for every positive integer n that is not a multiple of 3. [4]



2.4 Prove that the product of two odd integers is odd. [3]



Exercises 2.3·2.4

2.3 Prove by exhaustion that $n^2 - 1$ is divisible by 3 for every positive integer n that is not a multiple of 3. [4]

assume → deduce → conclude

2.3 Non-multiples of 3 are $n = 3q + 1$ or $n = 3q + 2$ (complete case list).

- $n = 3q + 1$: $n^2 - 1 = 9q^2 + 6q = 3(3q^2 + 2q)$ ✓
- $n = 3q + 2$: $n^2 - 1 = 9q^2 + 12q + 3 = 3(3q^2 + 4q + 1)$ ✓ Both cases give a multiple of 3. ■

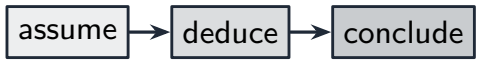
2.4 Prove that the product of two odd integers is odd. [3]

assume → deduce → conclude

2.4 Let the integers be $2a+1$ and $2b+1$: their product is $4ab+2a+2b+1 = 2(2ab+a+b)+1$, which is one more than an even number – odd. ■

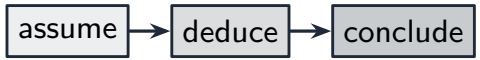
Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q10.)* Use algebra to prove by exhaustion that for all positive integers m that are not multiples of 3, the value of $m^2 + 3m + 2$ is always a multiple of 3. [4]



Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q10.)* Use algebra to prove by exhaustion that for all positive integers m that are not multiples of 3, the value of $m^2 + 3m + 2$ is always a multiple of 3. [4]

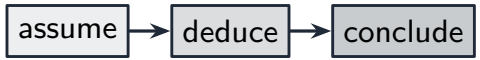


3.1 $m^2 + 3m + 2 = (m+1)(m+2)$. Cases: $m = 3q+1$ or $m = 3q+2$.

- $m = 3q+1$: $(3q+2)(3q+3) = 3(3q+2)(q+1)$ – multiple of 3.
- $m = 3q+2$: $(3q+3)(3q+4) = 3(q+1)(3q+4)$ – multiple of 3. Exhausting all non-multiples of 3, the expression is always a multiple of 3. ■ Check: $m = 1$: 6; $m = 2$: 12; $m = 4$: 30; $m = 5$: 42 – all divisible by 3 ✓.

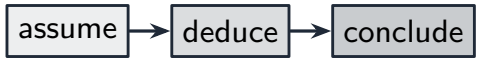
Exercise 3.2

3.2 Prove by exhaustion that no perfect square ends in the digit 8. [4]



Exercise 3.2

3.2 Prove by exhaustion that no perfect square ends in the digit 8. [4]



3.2 A square's last digit depends only on its root's last digit. Exhaust the ten endings:

$$0^2 \to 0,\ 1^2 \to 1,\ 2^2 \to 4,\ 3^2 \to 9,\ 4^2 \to 6,\ 5^2 \to 5,\ 6^2 \to 6,\ 7^2 \to 9,\ 8^2 \to 4,\ 9^2 \to 1$$

The possible endings are $0, 1, 4, 5, 6, 9$ – never 8. ■

Local review guidance: complete case structure or a valid counterexample; algebra carried correctly; the conclusion states what was proved, with the case list justified. This is authored classroom guidance.