Course notes

Pearson Edexcel · International A-Level · Mathematics

YMA01: course teaching notes

Version: Issue 3, April 2019; first teaching 2018; unit assessment from 2019

These are original teaching notes, and each lesson is labelled with its tier.

P1 · Indices, surds and standard form

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** 指数: The power to which a base is raised.

The method

- For the same positive base, multiplication adds indices and division subtracts them.
- A negative index means reciprocal; a fractional index represents a root.
- Standard form has 1≤a<10.

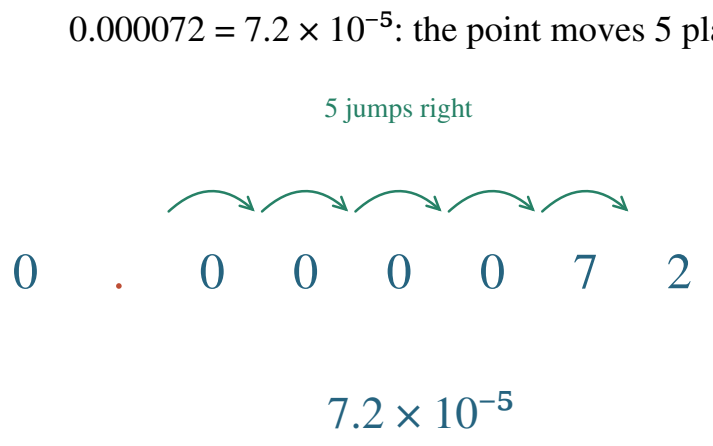
A worked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

■ Worked example

0.000072=7.2×10^(-5). Also 16^(3/4)=(16^(1/4))^3=2^3=8. Simplify √72=6√2, then rationalise 1/√2=√2/2.

exercises.lol exercises.com.cn



Where it goes wrong

- Index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.
- Do not round a surd when an exact answer is requested.

> **Watch out**
>
> **Not true:** For every positive a, a^2+a^3 equals a^5. Index laws only add powers when the terms are multiplied: $2^2+2^3=12$, but $2^5=32$.

Applying it

- Check powers of ten against the original quantity.
- Use surds for exact geometry, and round only the final length when the question asks for a decimal.

In your course

- Pearson Edexcel International A Level Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Index**: The power to which a base is raised.
- For the same positive base, multiplication adds indices and division subtracts them.
- Watch for: index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.

P1 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area x(10-x). What widths give at least 21 square metres?
- This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of ax^2+bx+c=0.

The method

- Factor where possible; otherwise complete the square or use the quadratic formula.
- A quadratic inequality needs the sign on intervals, not only the roots.
- The discriminant identifies repeated or missing real roots.

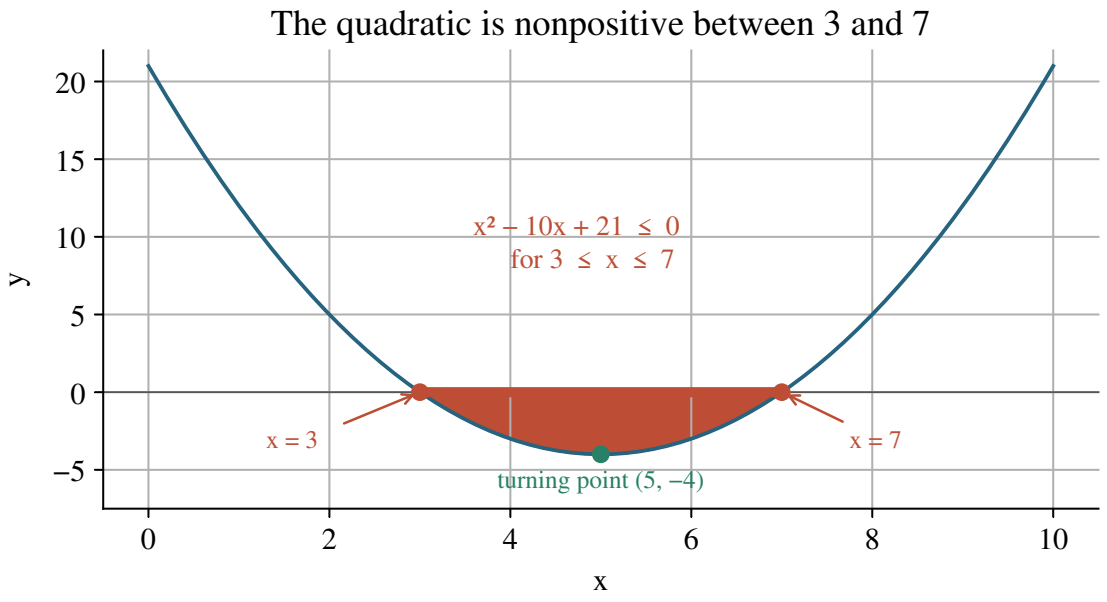
A worked case

$$ax^2 + bx + c = 0, \qquad \Delta = b^2 - 4ac$$

■ **Worked example**

The condition is x(10-x)≥21, so x^2-10x+21≤0. Factor (x-3)(x-7)≤0. The upward parabola is nonpositive between its roots, giving 3≤x≤7. The maximum area is 25 at x=5.

exercises.lol exercises.com.cn



Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

> ■ **Watch out**
>
> **Not true:** A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- Use the vertex to interpret an optimum.
- Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

In your course

- Pearson Edexcel International A Level Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Discriminant**: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.
- Factor where possible; otherwise complete the square or use the quadratic formula.
- Watch for: multiplying an inequality by a negative number reverses its direction.

P1 · Coordinate geometry and tangents

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient** 斜率: The change in y divided by the corresponding change in x.

The method

- A line through (x_1,y_1) with gradient m has $y-y_1=m(x-x_1)$.
- Parallel lines have equal gradients.
- Finite perpendicular gradients multiply to -1.
- A circle has $(x-a)^2+(y-b)^2=r^2$.

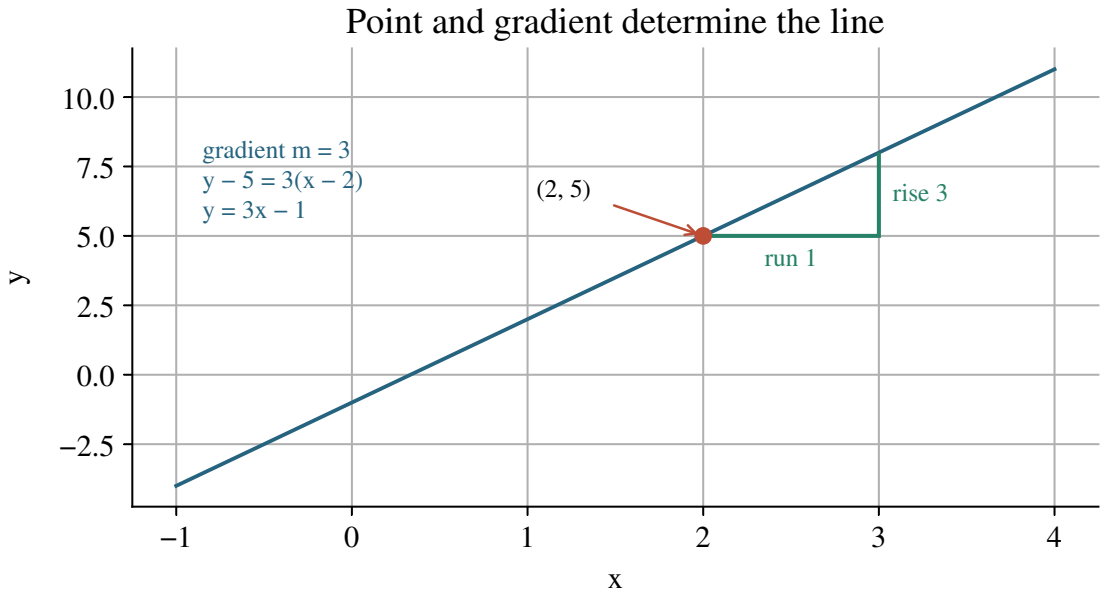
A worked case

$$y - y_1 = m(x - x_1), \qquad (x - a)^2 + (y - b)^2 = r^2$$

■ **Worked example**

Through (2,5) with gradient 3, y-5=3(x-2), so y=3x-1. A perpendicular through the same point has y-5=-(x-2)/3. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.

exercises.lol exercises.com.cn



Where it goes wrong

- A vertical line has no finite gradient; do not force it into y=mx+c.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

> **Watch out**
>
> **Not true:** Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to -1: a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

Applying it

- Before solving a line-circle intersection, predict whether there are zero, one or two intersections.
- Substitution produces a quadratic whose discriminant checks the prediction.

In your course

- Pearson Edexcel International A Level Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Gradient**: The change in y divided by the corresponding change in x.
- A line through (x_1,y_1) with gradient m has $y-y_1=m(x-x_1)$.
- Watch for: A vertical line has no finite gradient; do not force it into y=mx+c.

P1 · Right triangles and non-right triangles

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** 斜边: The side opposite the right angle in a right-angled triangle.

The method

- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- For other triangles, use the sine or cosine rule, or area=ab sin C/2.

A worked case

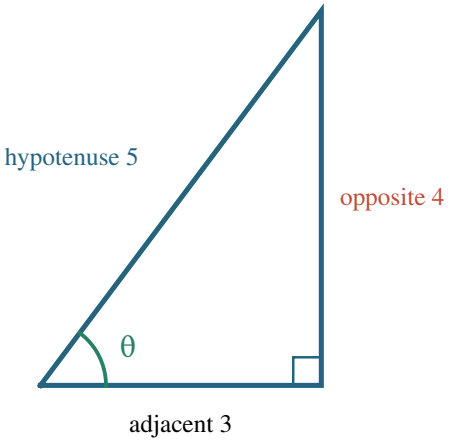
$$a^2 + b^2 = c^2, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

■ **Worked example**

The ladder length is $c=\sqrt{(3^2+4^2)}=5$ m. Its angle to the ground satisfies $\tan\theta=4/3$, so $\theta\approx53.1°$. With two sides 6 and 8 enclosing 60°, $c^2=6^2+8^2-2\times6\times8\cos60°=52$.

exercises.lol exercises.com.cn

Label the known sides before choosing a rule



opposite: across from θ
adjacent: next to θ
hypotenuse: across from the right angle

$\tan\theta = 4/3$ $\sin\theta = 4/5$
$\cos\theta = 3/5$

Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

■ Watch out

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
- Explain why the chosen triangle contains the required length or angle.

In your course

- Pearson Edexcel International A Level Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Hypotenuse**: The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- Watch for: label sides relative to the chosen angle.

P1 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.

The method

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
- Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve.
- List every solution in the stated interval.

A worked case

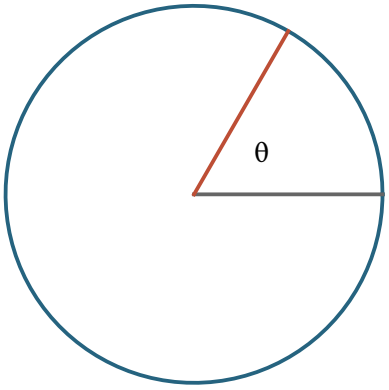
$$s = r\theta, \qquad A = \frac{1}{2}r^2\theta, \qquad \sin^2\theta + \cos^2\theta = 1$$

■ **Worked example**

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

exercises.lol exercises.com.cn

A radius fixes distance from the centre



radius × angle gives arc length

Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by sin x can discard solutions where sin x=0; check that case first.

> ■ **Watch out**
>
> **Not true:** The principal inverse sine value is the only solution in a full turn. Sine takes most values twice in a full turn: sin x=0.5 gives 30° and 150°.

Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

In your course

- Pearson Edexcel International A Level Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Radian**: The angle subtended by an arc whose length equals the radius.
- For θ in radians, arc length s=rθ and sector area A=r$^2\theta$/2.
- Watch for: A calculator's inverse sine gives one principal value.

P1 · Derivatives and stationary points

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.

The method

- For y=ax^n, dy/dx=anx^(n-1).
- A stationary point satisfies dy/dx=0.
- Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.

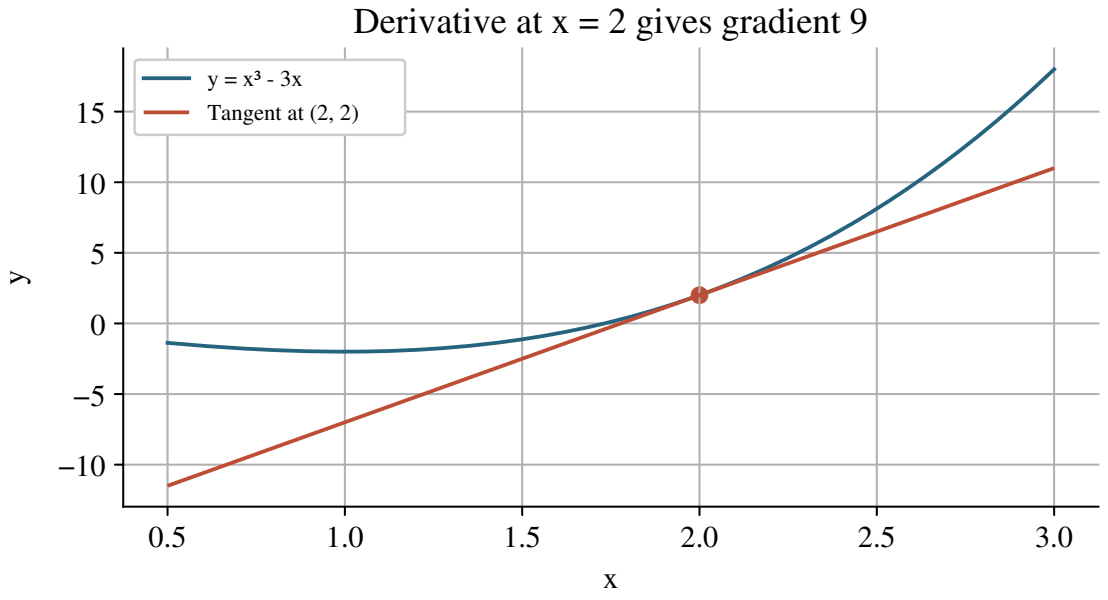
A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad m = f'(x_0)$$

■ **Worked example**

For y=x^3-3x, dy/dx=3x^2-3. At x=2, the gradient is 9 and y=2. The tangent is y-2=9(x-2). The normal gradient is -1/9.

exercises.lol exercises.com.cn



Where it goes wrong

- A derivative gives a gradient, not the ordinate.
- Evaluate y and dy/dx separately at the supplied x-coordinate; the normal gradient is the negative reciprocal of a nonzero tangent gradient.

■ Watch out

Not true: A tangent and normal at the same point have the same gradient. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

Applying it

- P1 covers polynomial derivatives, gradients, tangents and normals.
- Optimization and stationary-point classification are taught in P2; they are not introduced here.

In your course

- Pearson Edexcel International A Level Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Derivative**: The instantaneous rate of change, also the gradient of a tangent.
- For y=ax^n, dy/dx=anx^(n-1).
- Watch for: A derivative gives a gradient, not the ordinate.

P1 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.

The method

- Integrate polynomial powers by increasing the index by one and dividing by the new index.
- Include an arbitrary constant and use a supplied point to determine it.

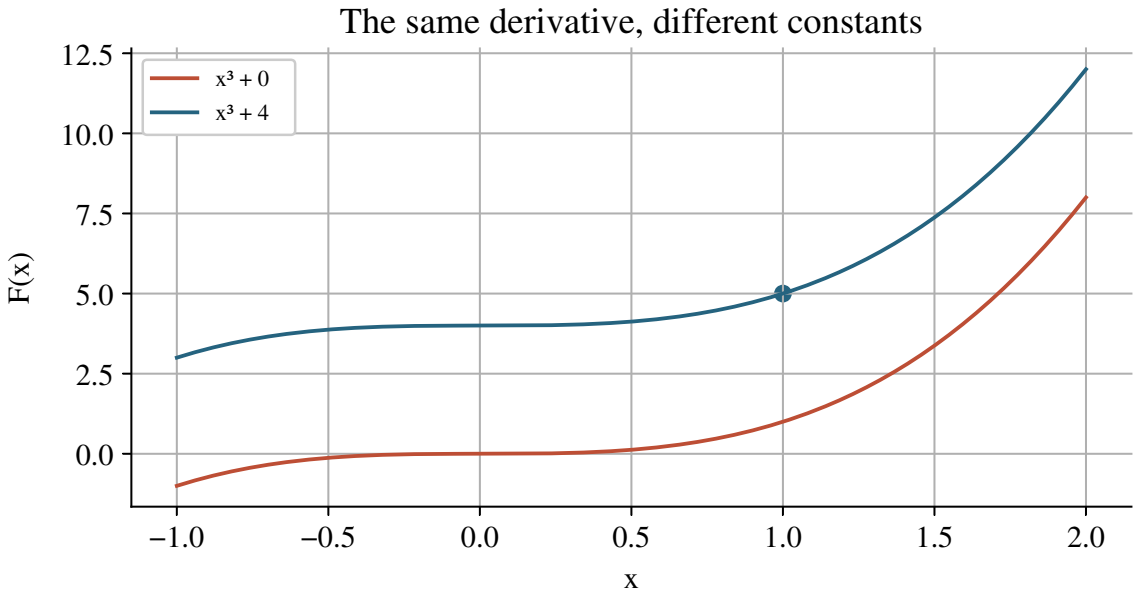
A worked case

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

■ **Worked example**

An antiderivative of $3x^2$ is x^3+C because differentiating x^3 gives $3x^2$. If $F(1)=5$, then $1+C=5$, so $C=4$ and $F(x)=x^3+4$.

exercises.lol exercises.com.cn



Where it goes wrong

- An indefinite integral needs a constant.
- Integrating each term changes its power; copying the derivative rule gives the wrong result.

> **■ Watch out**
>
> **Not true:** Every indefinite integral has only one possible antiderivative. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

Applying it

- P1 uses indefinite polynomial integration and a point to determine the constant.
- Definite integrals, areas and the trapezium rule are P2 content.

In your course

- Pearson Edexcel International A Level Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Antiderivative**: A function whose derivative equals the given integrand.
- Integrate polynomial powers by increasing the index by one and dividing by the new index.
- Watch for: an indefinite integral needs a constant.

P2 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
- This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.

The method

- A deductive proof starts from stated definitions or assumptions.
- A counterexample refutes an all-values claim.
- Induction requires a base case and an implication from n=k to n=k+1; checking successive cases alone is not induction.

A worked case

$$(2a+1)+(2b+1)=2(a+b+1)$$

■ **Worked example**

For odd integers 2a+1 and 2b+1, their sum is 2(a+b+1), hence even. The claim n^2+n+41 is always prime fails at n=41: the value is 41×43=1763. For 1+...+n=n(n+1)/2, the induction step adds k+1 to the assumed sum.

exercises.lol exercises.com.cn

Odd + odd: (2a + 1) + (2b + 1)

Factor 2: 2(a + b + 1)

The bracket is an integer ⇒ the sum is even

Where it goes wrong

- Do not assume the conclusion while proving it.
- In contradiction, identify the impossible consequence and reject the initial contrary assumption.
- Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.

> ■ **Watch out**
>
> **Not true:** Checking the first ten integers proves a claim for every positive integer. A pattern can hold for the first ten cases and then fail; a proof must cover every case.

Applying it

- AQA proof includes deduction, exhaustion and contradiction.
- IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction.
- Keep the method matched to the named unit.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Counterexample**: A single valid case that disproves a universal claim.
- A deductive proof starts from stated definitions or assumptions.
- Watch for: do not assume the conclusion while proving it.

P2 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area x(10-x). What widths give at least 21 square metres?
- This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of ax^2+bx+c=0.

The method

- Factor where possible; otherwise complete the square or use the quadratic formula.
- A quadratic inequality needs the sign on intervals, not only the roots.
- The discriminant identifies repeated or missing real roots.

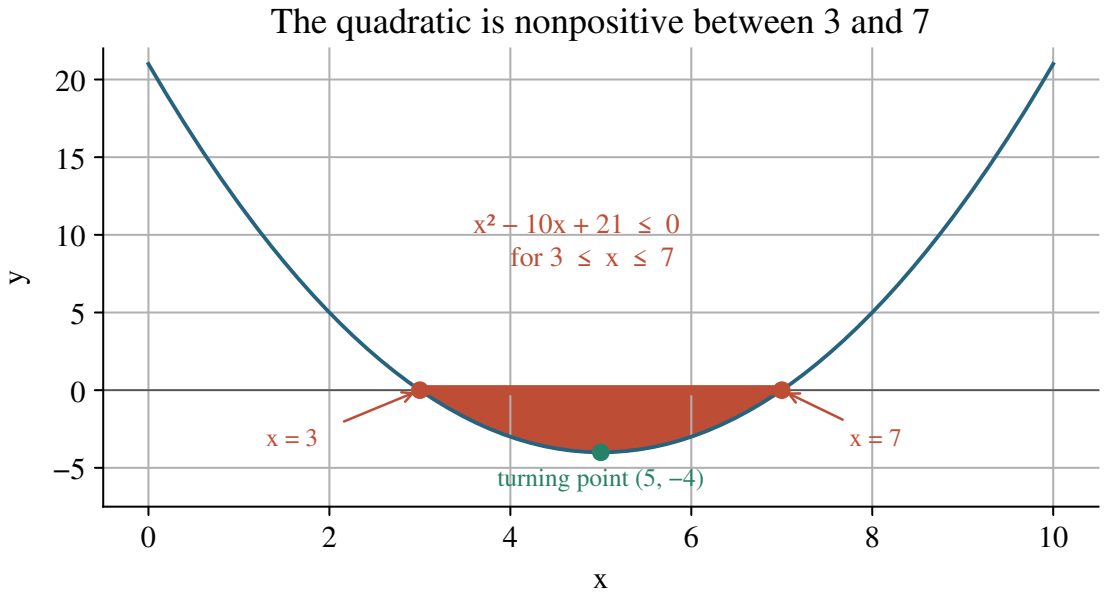
A worked case

$$ax^2 + bx + c = 0, \qquad \Delta = b^2 - 4ac$$

Worked example

The condition is x(10-x)≥21, so x^2-10x+21≤0. Factor (x-3)(x-7)≤0. The upward parabola is nonpositive between its roots, giving 3≤x≤7. The maximum area is 25 at x=5.

exercises.lol exercises.com.cn



Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

> **Watch out**
>
> **Not true:** A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- Use the vertex to interpret an optimum.
- Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

In your course

- Pearson Edexcel International A Level Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Discriminant**: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.
- Factor where possible; otherwise complete the square or use the quadratic formula.
- Watch for: multiplying an inequality by a negative number reverses its direction.

P2 · Coordinate geometry and tangents

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient** 斜率: The change in y divided by the corresponding change in x.

The method

- A line through (x_1,y_1) with gradient m has $y-y_1=m(x-x_1)$.
- Parallel lines have equal gradients.
- Finite perpendicular gradients multiply to -1.
- A circle has $(x-a)^2+(y-b)^2=r^2$.

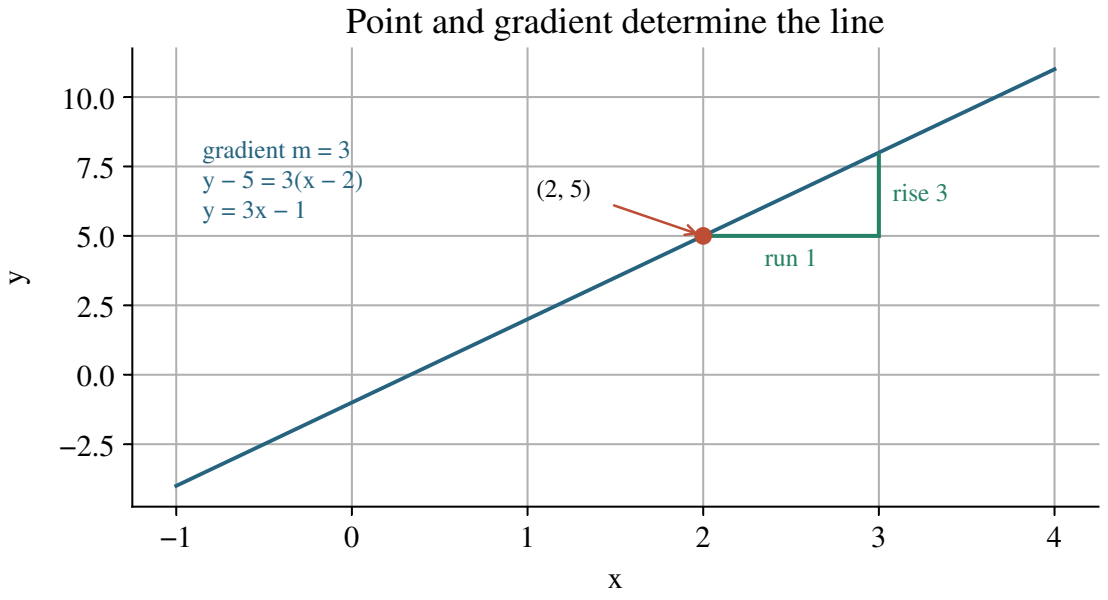
A worked case

$$y - y_1 = m(x - x_1), \qquad (x - a)^2 + (y - b)^2 = r^2$$

■ **Worked example**

Through (2,5) with gradient 3, y-5=3(x-2), so y=3x-1. A perpendicular through the same point has y-5=-(x-2)/3. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.

exercises.lol exercises.com.cn



Where it goes wrong

- A vertical line has no finite gradient; do not force it into y=mx+c.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

■ Watch out

Not true: Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to -1: a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

Applying it

- Before solving a line-circle intersection, predict whether there are zero, one or two intersections.
- Substitution produces a quadratic whose discriminant checks the prediction.

In your course

- Pearson Edexcel International A Level Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ Key idea

- **Gradient**: The change in y divided by the corresponding change in x.
- A line through (x_1,y_1) with gradient m has $y-y_1=m(x-x_1)$.
- Watch for: A vertical line has no finite gradient; do not force it into y=mx+c.

P2 · Sequences, series and recurrence

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio** 公比: The constant multiplier between consecutive terms of a geometric sequence.

The method

- For an arithmetic progression, u_n=a+(n-1)d and S_n=n(2a+(n-1)d)/2.
- For a geometric progression, u_n=ar^(n-1) and S_n=a(1-r^n)/(1-r).
- An infinite geometric sum exists only if |r|<1.

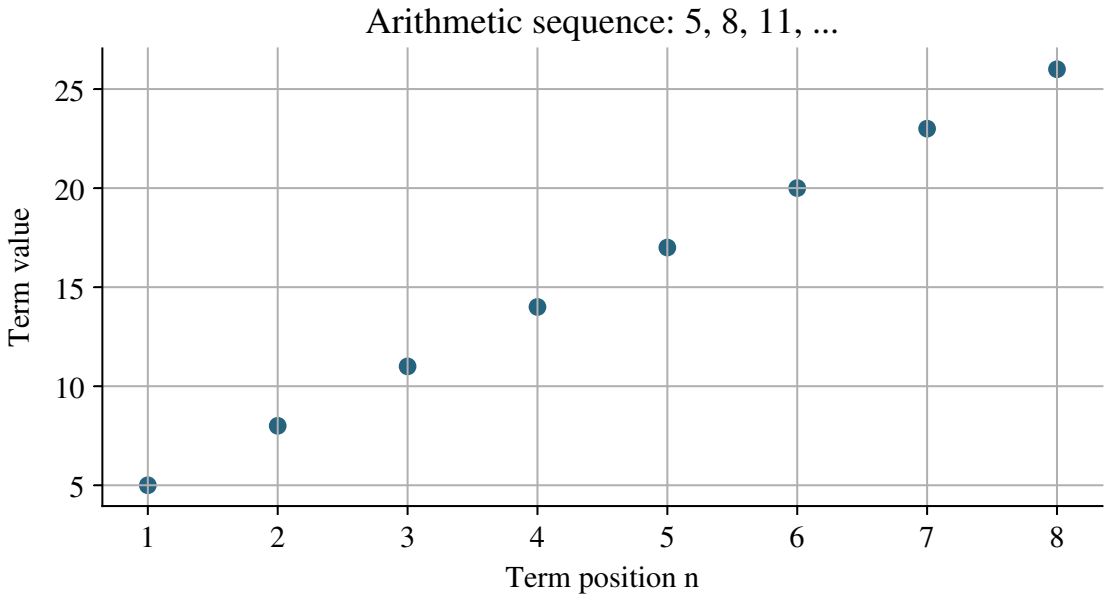
A worked case

$$u_n = a + (n-1)d, \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

■ Worked example

For a=5,d=3,n=8, u_8=5+7×3=26 and S_8=8(10+21)/2=124. For a=12,r=1/2, S infinity=12/(1-1/2)=24. For u_(n+1)=2u_n+1 with u_1=1, the next terms are 3,7,15.

exercises.lol exercises.com.cn



Where it goes wrong

- The first term has index 1, so the exponent is n-1.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

> ■ **Watch out**
>
> **Not true:** Every geometric series has a finite sum to infinity. A sum to infinity exists only when $-1<r<1$; for $r=2$ the terms grow without limit.

Applying it

- Explain whether the context justifies additive or multiplicative change.
- In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.

In your course

- Pearson Edexcel International A Level Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Common ratio**: The constant multiplier between consecutive terms of a geometric sequence.
- For an arithmetic progression, u_n=a+(n-1)d and S_n=n(2a+(n-1)d)/2.
- Watch for: the first term has index 1, so the exponent is n-1.

P2 · Binomial expansion and valid approximations

How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** 二项式系数: The number of ways to choose a specified number of objects from a set.

The method

- For positive integer n, (a+b)^n is a finite binomial expansion.
- For noninteger n, expand (1+x)^n as 1+nx+n(n-1)x^2/2+... with |x|<1.
- Factor out constants before using this form.

A worked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$$

■ **Worked example**

(1+2x)^5=1+10x+40x^2+80x^3+80x^4+32x^5. For (1+x)^(-1), the first three terms are 1-x+x^2; at x=0.1 this gives 0.91 versus the exact 1/1.1≈0.909091.

exercises.lol exercises.com.cn

$(1 + 2x)^5$: Pascal row × powers of 2

Pascal row 5	1	5	10	10	5	1
$\times 2^k$	1	2	4	8	16	32
coefficient	**1**	**10**	**40**	**80**	**80**	**32**
	1	x	x^2	x^3	x^4	x^5

Where it goes wrong

- An expansion in 2x requires $|2x|<1$ for the infinite series, not merely $|x|<1$.
- Positive-integer expansions are finite and do not have that convergence restriction.

> ■ **Watch out**
>
> **Not true:** Every binomial expansion is a finite polynomial, including powers that are not positive integers. For a negative or fractional power the expansion is an infinite series, valid only for $|x|<1$.

Applying it

- State the range of validity alongside an approximation.
- Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x.

In your course

- Pearson Edexcel International A Level Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Binomial coefficient**: The number of ways to choose a specified number of objects from a set.
- For positive integer n, (a+b)^n is a finite binomial expansion.
- Watch for: an expansion in 2x requires |2x|<1 for the infinite series, not merely |x|<1.

P2 · Exponentials, logarithms and modelling

Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** 半衰期: The time for a decaying quantity to fall to half its initial value.

The method

- For y=Ae^(kt), k is a proportional rate.
- Taking logs gives ln y=ln A+kt.
- Logarithms require positive arguments, and log(x+y) is not log x+log y.

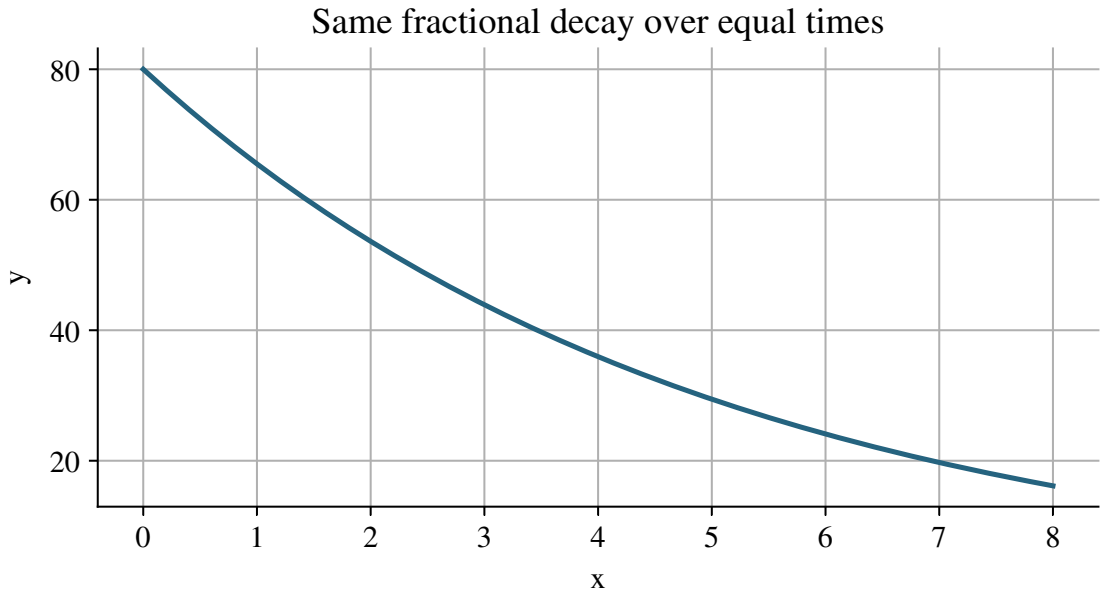
A worked case

$$y = Ae^{kt}, \qquad \ln y = \ln A + kt$$

■ **Worked example**

If y=80e^(-0.2t), y=40 gives e^(-0.2t)=1/2. Hence t=ln2/0.2≈3.466. For 3^x=20, x=ln20/ln3. Plotting ln y against t linearises this exponential model.

exercises.lol exercises.com.cn



Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative k describes decay; a negative starting amount usually contradicts the context.

Watch out

Not true: For positive x and y, $\ln(x+y)=\ln x+\ln y$. The log of a sum has no simple law; the product rule is $\ln(xy)=\ln x+\ln y$.

Applying it

- Compare actual observations with the model.
- Systematic departures may indicate changing conditions.
- In a report, explain what the rate and initial value mean, rather than giving a bare equation.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Half-life**: The time for a decaying quantity to fall to half its initial value.
- For y=Ae^(kt), k is a proportional rate.
- Watch for: A fitted exponential is a model, not a guarantee.

P2 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.

The method

- For θ in radians, arc length s=rθ and sector area A=r$^2\theta$/2.
- Use sin$^2\theta$+cos$^2\theta$=1 and angle identities to simplify or solve.
- List every solution in the stated interval.

A worked case

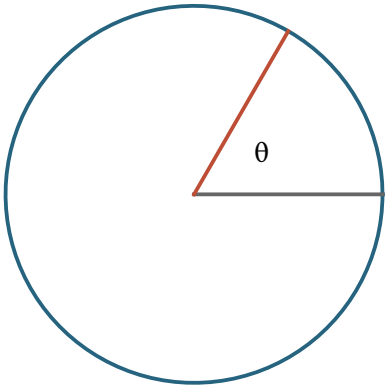
$$s = r\theta, \qquad A = \frac{1}{2}r^2\theta, \qquad \sin^2\theta + \cos^2\theta = 1$$

■ Worked example

For r=4 and θ=π/3, s=4π/3 and A=8π/3. Solving sin x=1/2 on 0≤x<2π gives x=π/6 and 5π/6. For sin2x=1/2, solve first on 0≤2x<4π before dividing by 2.

exercises.lol exercises.com.cn

A radius fixes distance from the centre



radius × angle gives arc length

Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by sin x can discard solutions where sin x=0; check that case first.

> ■ **Watch out**
>
> **Not true:** The principal inverse sine value is the only solution in a full turn. Sine takes most values twice in a full turn: sin x=0.5 gives 30° and 150°.

Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

In your course

- Pearson Edexcel International A Level Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Radian**: The angle subtended by an arc whose length equals the radius.
- For θ in radians, arc length s=rθ and sector area A=r$^2\theta$/2.
- Watch for: A calculator's inverse sine gives one principal value.

P2 · Derivatives and stationary points

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.

The method

- For y=ax^n, dy/dx=anx^(n-1).
- A stationary point satisfies dy/dx=0.
- Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.

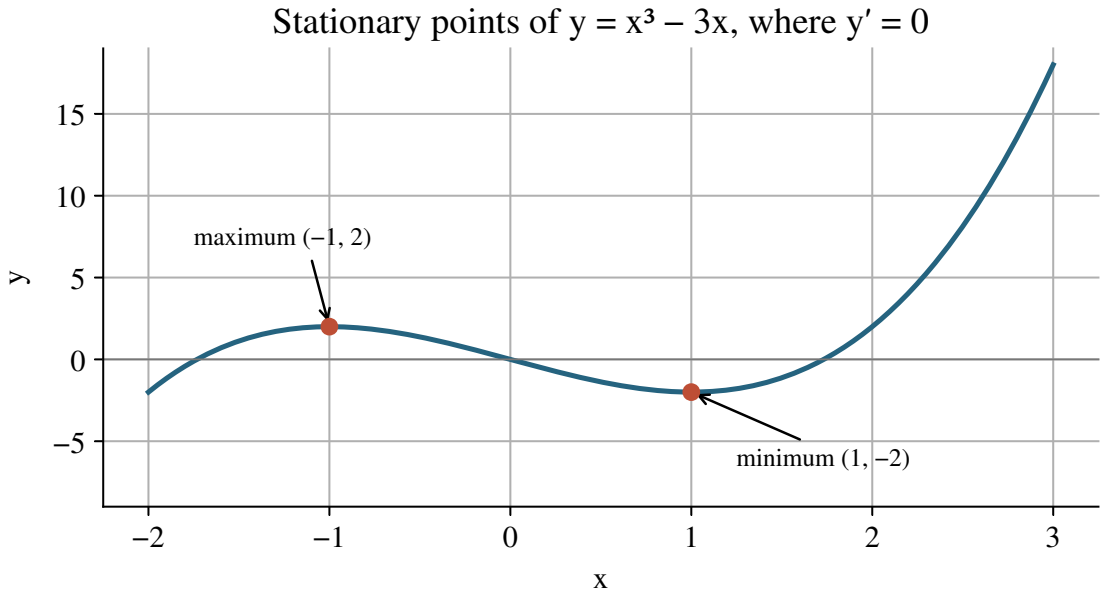
A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \qquad f'(x) = 0$$

■ **Worked example**

For y=x^3-3x, dy/dx=3x^2-3. At x=1, the gradient is 0 and y=-2. The second derivative is 6x, positive at x=1, so this is a local minimum. At x=-1, y=2 and the second derivative is negative, giving a local maximum.

exercises.lol exercises.com.cn



Where it goes wrong

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.
- An endpoint can also produce an extreme value on a restricted domain.

> ■ **Watch out**
>
> **Not true:** Every point with zero derivative is a local maximum or minimum. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

Applying it

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there.
- At advanced level, connect the derivative to rates and optimization with a valid domain.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Derivative**: The instantaneous rate of change, also the gradient of a tangent.
- For y=ax^n, dy/dx=anx^(n-1).
- Watch for: A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.

P2 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.

The method

- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- A definite integral is signed accumulation.
- Split at sign changes when total area or distance is required.

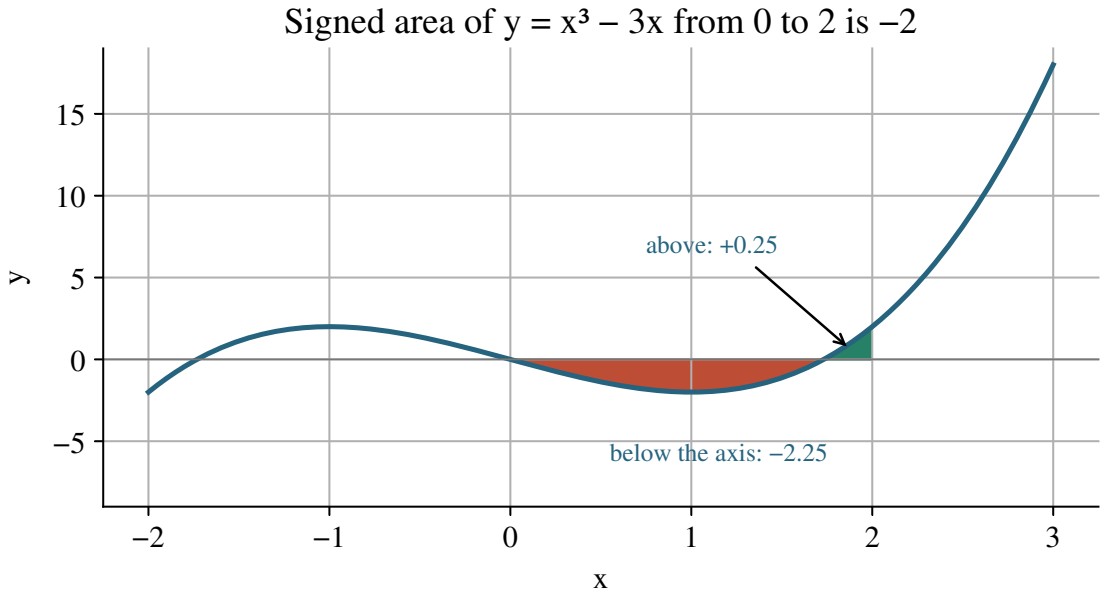
A worked case

$$\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

■ Worked example

For v(t)=3t^2-3 over 0≤t≤2, displacement=[t^3-3t]_0^2=2. Since v changes sign at t=1, distance=-[t^3-3t]_0^1+[t^3-3t]_1^2=2+4=6. The constants cancel only for a definite integral.

exercises.lol exercises.com.cn



Where it goes wrong

- The integral of 1/x is ln|x|+C, not the power rule with n=-1.
- Signed area can be zero even when the total enclosed area is positive.

> ■ **Watch out**
>
> **Not true:** A definite integral equals the total positive area, even when the graph crosses the axis. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

Applying it

- Identify what the integral means and include the correct units.
- A rate measured per second integrates to the underlying quantity, not to another rate.
- Differentiate an antiderivative to check it.

In your course

- Pearson Edexcel International A Level Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Antiderivative**: A function whose derivative equals the given integrand.
- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- Watch for: the integral of 1/x is ln|x|+C, not the power rule with n=-1.

P2 · Root finding and numerical integration

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
- This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.

The method

- For equal strip width h, the trapezium estimate is h/2 times the sum of the two endpoint heights plus twice the internal heights.

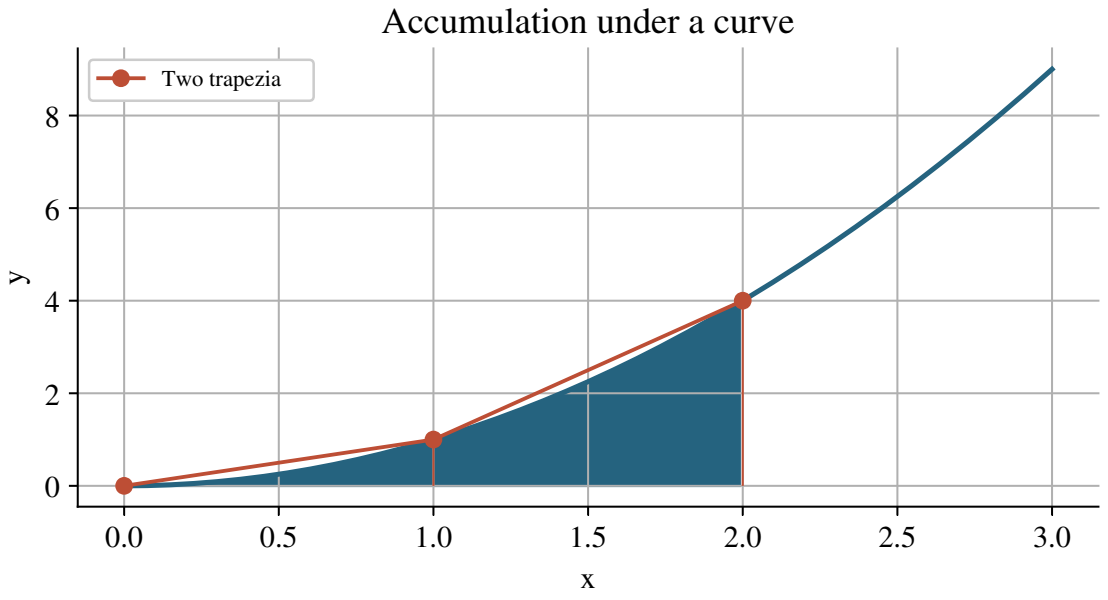
A worked case

$$A \approx \frac{h}{2}(y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n)$$

■ **Worked example**

For $y=x^2$ on [0,2] with two equal strips, h=1 and the heights are 0,1,4. The estimate is $(1/2)(0+2\times1+4)=3$. The exact area is 8/3, so this convex curve gives an overestimate.

exercises.lol exercises.com.cn



Where it goes wrong

- Use equal strip widths and include internal heights twice.
- Curvature decides whether the estimate is above or below the exact area; being increasing alone does not.

■ Watch out

Not true: An increasing function gives a trapezium overestimate. A sign change shows a root only if the function is continuous there: 1/x changes sign across x=0 but has no root.

Applying it

- P2 introduces the trapezium rule.
- Newton–Raphson and other root-finding methods belong to other named units; they are not part of this P2 lesson.

In your course

- Pearson Edexcel International A Level Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Iteration**: A repeated update in which each new approximation is calculated from the previous one.
- For equal strip width h, the trapezium estimate is h/2 times the sum of the two endpoint heights plus twice the internal heights.
- Watch for: use equal strip widths and include internal heights twice.

P3 · Domains, inverses and composition

Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** 定义域: The set of allowed inputs to a function.

The method

- State the domain and range.
- For an inverse, first ensure the function is one-to-one on its domain.
- Composition fg means apply g first, then f; the intermediate output must be an allowed input to f.

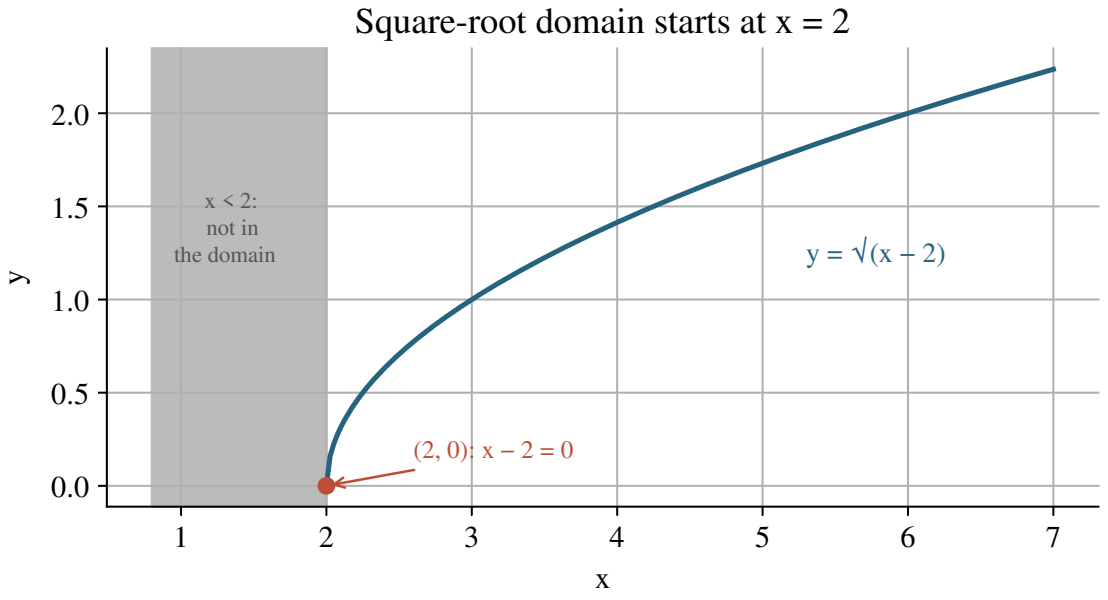
A worked case

$$f(g(x)) = (f \circ g)(x), \qquad f^{-1}(f(x)) = x$$

■ **Worked example**

For f(x)=√(x-2), x≥2 and the range is y≥0. From y=√(x-2), x=y^2+2. Thus f inverse(x)=x^2+2 with x≥0. For g(x)=x+3, fg(1)=f(4)=√2.

exercises.lol exercises.com.cn



Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside f has the opposite sign to the graph's movement.

> ■ **Watch out**
>
> **Not true:** Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one (f(1)=f(−1) for x^2), so its domain must be restricted first.

Applying it

- Check f(f inverse(x))=x on the inverse domain.
- Use a sketch to test whether a horizontal line meets the original graph more than once.

In your course

- Pearson Edexcel International A Level Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Domain**: The set of allowed inputs to a function.
- State the domain and range.
- Watch for: squaring can introduce extraneous solutions.

P3 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.

The method

- For θ in radians, arc length s=rθ and sector area A=r$^2\theta/2$.
- Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve.
- List every solution in the stated interval.

A worked case

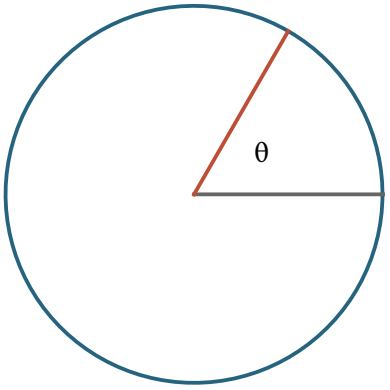
$$s = r\theta, \qquad A = \frac{1}{2}r^2\theta, \qquad \sin^2\theta + \cos^2\theta = 1$$

■ **Worked example**

For r=4 and $\theta=\pi/3$, s=$4\pi/3$ and A=$8\pi/3$. Solving sin x=1/2 on $0\leq$x$<2\pi$ gives x=$\pi/6$ and $5\pi/6$. For sin2x=1/2, solve first on $0\leq$2x$<4\pi$ before dividing by 2.

exercises.lol exercises.com.cn

A radius fixes distance from the centre



radius × angle gives arc length

Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by sin x can discard solutions where sin x=0; check that case first.

> ■ **Watch out**
>
> **Not true:** The principal inverse sine value is the only solution in a full turn. Sine takes most values twice in a full turn: sin x=0.5 gives 30° and 150°.

Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

In your course

- Pearson Edexcel International A Level Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Radian**: The angle subtended by an arc whose length equals the radius.
- For θ in radians, arc length s=rθ and sector area A=r$^2\theta$/2.
- Watch for: A calculator's inverse sine gives one principal value.

P3 · Exponentials, logarithms and modelling

Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** 半衰期: The time for a decaying quantity to fall to half its initial value.

The method

- For y=Ae^(kt), k is a proportional rate.
- Taking logs gives ln y=ln A+kt.
- Logarithms require positive arguments, and log(x+y) is not log x+log y.

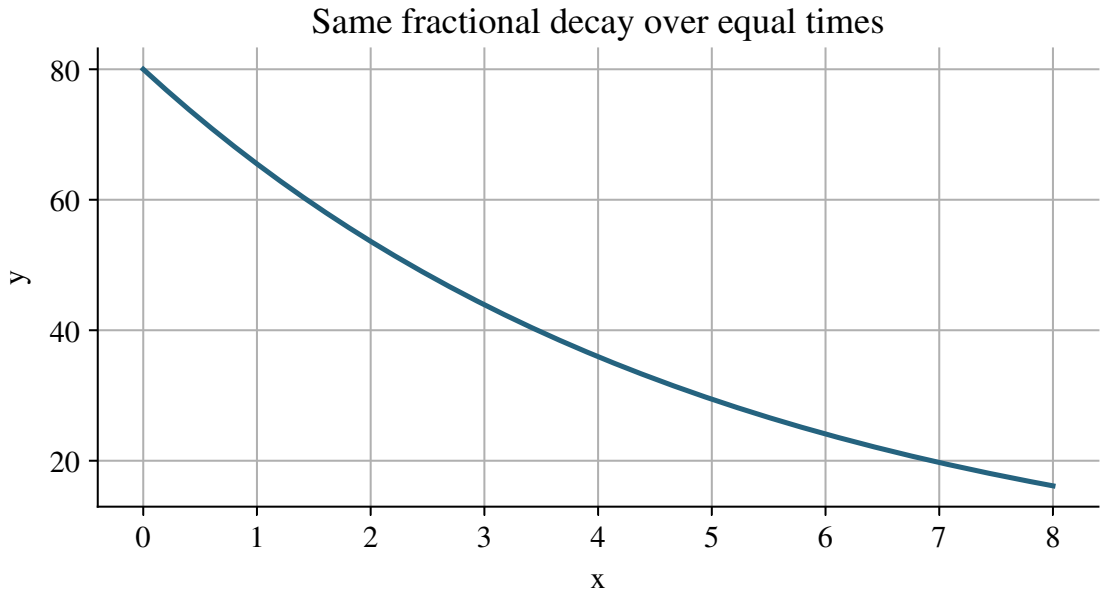
A worked case

$$y = Ae^{kt}, \qquad \ln y = \ln A + kt$$

■ **Worked example**

If y=80e^(-0.2t), y=40 gives e^(-0.2t)=1/2. Hence t=ln2/0.2≈3.466. For 3^x=20, x=ln20/ln3. Plotting ln y against t linearises this exponential model.

exercises.lol exercises.com.cn



Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative k describes decay; a negative starting amount usually contradicts the context.

Watch out

Not true: For positive x and y, $\ln(x+y)=\ln x+\ln y$. The log of a sum has no simple law; the product rule is $\ln(xy)=\ln x+\ln y$.

Applying it

- Compare actual observations with the model.
- Systematic departures may indicate changing conditions.
- In a report, explain what the rate and initial value mean, rather than giving a bare equation.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Half-life**: The time for a decaying quantity to fall to half its initial value.
- For y=Ae^(kt), k is a proportional rate.
- Watch for: A fitted exponential is a model, not a guarantee.

P3 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
- This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.

The method

- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- For uv, differentiate to u prime v+uv prime.
- For u/v, use (u prime v-uv prime)/v^2, where v≠0.

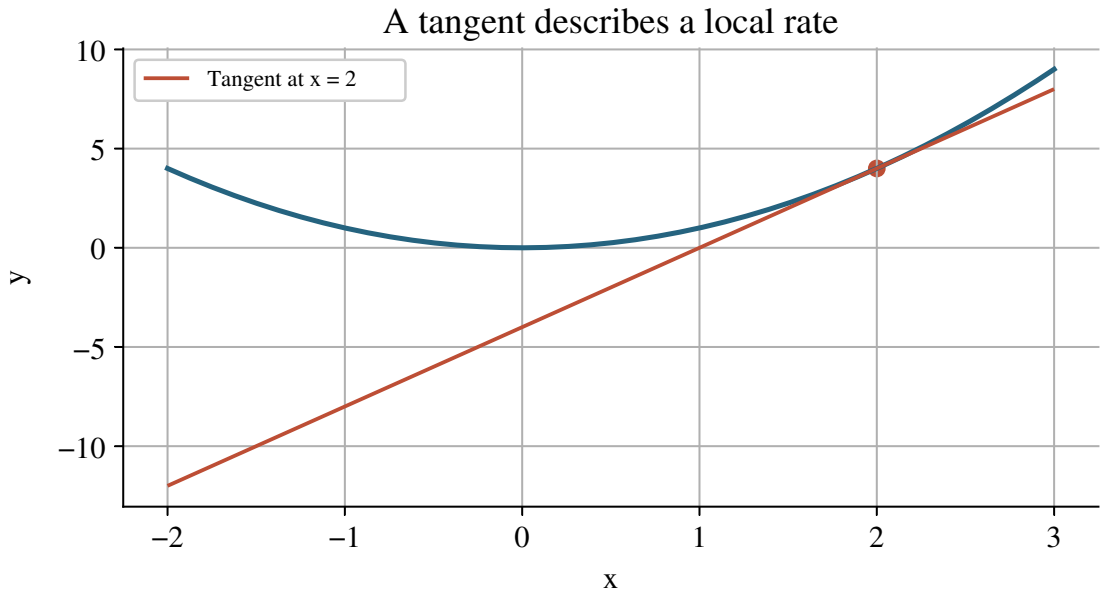
A worked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

■ Worked example

For y=(3x+1)^4, y prime=4(3x+1)^3×3=12(3x+1)^3. At x=0, the gradient is 12. For $x^2+y^2=25$, differentiate implicitly: 2x+2y y prime=0, so y prime=-x/y when y≠0.

exercises.lol exercises.com.cn



Where it goes wrong

- A derivative of a product is not the product of the derivatives.
- In implicit differentiation, every differentiated function of y brings a dy/dx factor.
- A quotient's denominator is squared.

> ■ **Watch out**
>
> **Not true:** The derivative of u(x)v(x) is u prime times v prime. The product rule is $(uv)'=u'v+uv'$: for $x \cdot x$ the derivative is 2x, not $1\times1=1$.

Applying it

- Choose a useful form before differentiating: expanding a short polynomial may be simpler.
- For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.

In your course

- Pearson Edexcel International A Level Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Chain rule**: The rule that multiplies the outer derivative by the inner derivative for a composite function.
- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- Watch for: A derivative of a product is not the product of the derivatives.

P3 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.

The method

- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- A definite integral is signed accumulation.
- Split at sign changes when total area or distance is required.

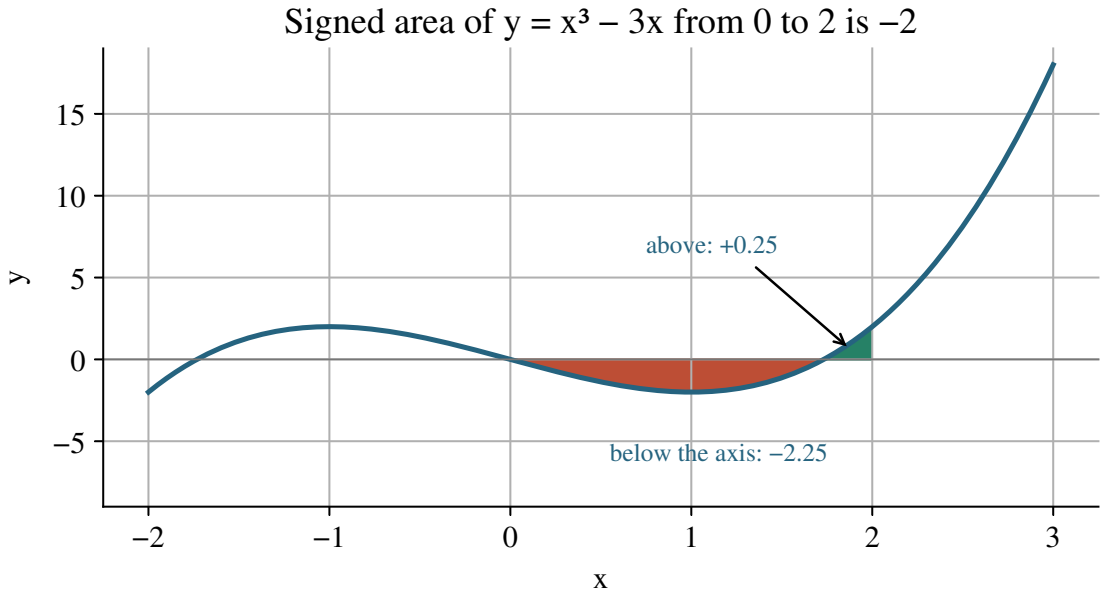
A worked case

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

■ **Worked example**

For v(t)=3t^2-3 over 0≤t≤2, displacement=[t^3-3t]_0^2=2. Since v changes sign at t=1, distance=-[t^3-3t]_0^1+[t^3-3t]_1^2=2+4=6. The constants cancel only for a definite integral.

exercises.lol exercises.com.cn



Where it goes wrong

- The integral of 1/x is ln|x|+C, not the power rule with n=-1.
- Signed area can be zero even when the total enclosed area is positive.

> ■ **Watch out**
>
> **Not true:** A definite integral equals the total positive area, even when the graph crosses the axis. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

Applying it

- Identify what the integral means and include the correct units.
- A rate measured per second integrates to the underlying quantity, not to another rate.
- Differentiate an antiderivative to check it.

In your course

- Pearson Edexcel International A Level Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Antiderivative**: A function whose derivative equals the given integrand.
- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- Watch for: the integral of 1/x is ln|x|+C, not the power rule with n=-1.

P3 · Fixed-point iteration

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
- This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.

The method

- Rearrange the equation as x=g(x), choose a starting value and iterate.
- A limit L must satisfy L=g(L).
- Near a fixed point, |g prime(L)|<1 provides a local convergence check; a sign bracket can validate the reported rounded root.

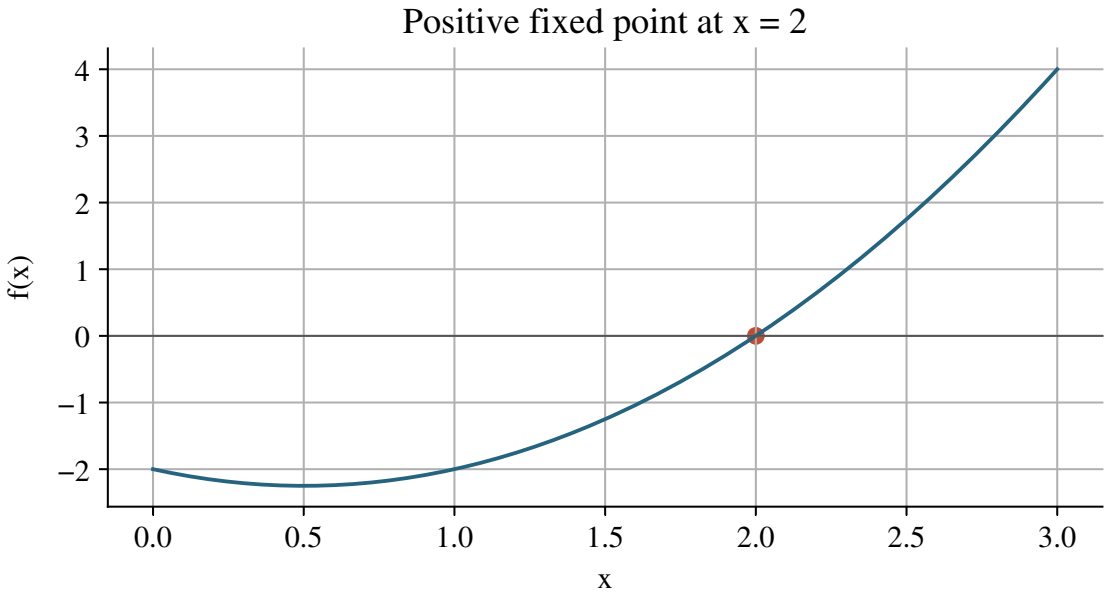
A worked case

$$x_{n+1} = \sqrt{2 + x_n}$$

■ **Worked example**

For x^2-x-2=0 choose x next=√(2+x), starting at x_0=1. Then x_1=√3≈1.73205, x_2≈1.93185 and the positive fixed point is 2. At 2, g prime=1/(2√4)=1/4, so small local errors shrink. The square-root rearrangement seeks the positive root; it does not give the negative root -1.

exercises.lol exercises.com.cn



Where it goes wrong

- Different rearrangements can have different convergence behaviour.
- Check the iterates, domain and original equation; a square-root update cannot reach a negative fixed point.

> ■ **Watch out**
>
> **Not true:** Every rearrangement of an equation converges to the same root from every starting value. A sign change shows a root only if the function is continuous there: 1/x changes sign across x=0 but has no root.

Applying it

- P3 uses equation rearrangement and numerical iteration.
- This lesson excludes Newton and numerical integration, which belong to other units.

In your course

- Pearson Edexcel International A Level Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Iteration**: A repeated update in which each new approximation is calculated from the previous one.
- Rearrange the equation as x=g(x), choose a starting value and iterate.
- Watch for: different rearrangements can have different convergence behaviour.

P4 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
- This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.

The method

- A deductive proof starts from stated definitions or assumptions.
- A counterexample refutes an all-values claim.
- Induction requires a base case and an implication from n=k to n=k+1; checking successive cases alone is not induction.

A worked case

$$(2a+1)+(2b+1)=2(a+b+1)$$

■ **Worked example**

For odd integers 2a+1 and 2b+1, their sum is 2(a+b+1), hence even. The claim n^2+n+41 is always prime fails at n=41: the value is 41×43=1763. For 1+...+n=n(n+1)/2, the induction step adds k+1 to the assumed sum.

exercises.lol exercises.com.cn

Odd + odd: (2a + 1) + (2b + 1)

Factor 2: 2(a + b + 1)

The bracket is an integer → the sum is even

Where it goes wrong

- Do not assume the conclusion while proving it.
- In contradiction, identify the impossible consequence and reject the initial contrary assumption.
- Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.

> ■ **Watch out**
>
> **Not true:** Checking the first ten integers proves a claim for every positive integer. A pattern can hold for the first ten cases and then fail; a proof must cover every case.

Applying it

- AQA proof includes deduction, exhaustion and contradiction.
- IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction.
- Keep the method matched to the named unit.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Counterexample**: A single valid case that disproves a universal claim.
- A deductive proof starts from stated definitions or assumptions.
- Watch for: do not assume the conclusion while proving it.

P4 · Equations, identities and rearrangement

When do two plans cost the same?

- Two mobile plans cost 20+3x and 44+x yuan for x GB. When do they cost the same?
- This lesson studies **identity** 恒等式: An equality that holds for every allowed value of its variable.

The method

- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Preserve equality by applying the same operation to both sides.
- State restrictions before dividing by a variable.

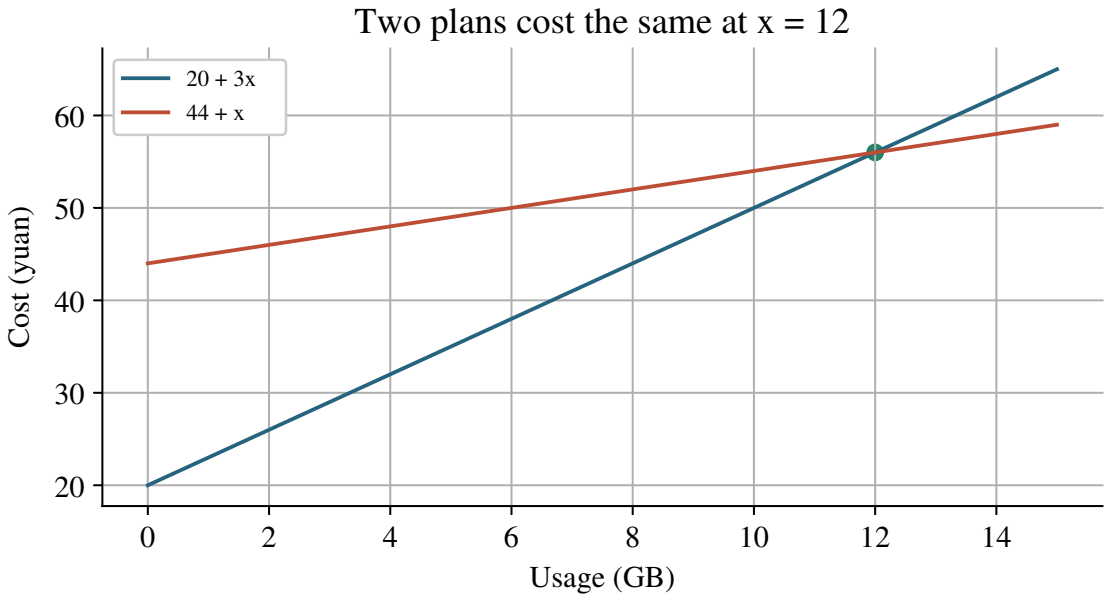
A worked case

$$C_1 = 20 + 3x, \qquad C_2 = 44 + x, \qquad C_1 = C_2$$

■ Worked example

20+3x=44+x gives 2x=24 and x=12. Both plans then cost 56. In $A=\pi r^2$, divide by π and take the positive square root to obtain $r=\sqrt{(A/\pi)}$, because r is a length.

exercises.lol exercises.com.cn



Where it goes wrong

- Cancelling a term is not the same as cancelling a factor.
- In $(x^2+2x)/x$, factor the numerator and retain $x \neq 0$.
- Check a rearrangement by substitution.

> ■ **Watch out**
>
> **Not true:** Cancelling x from $(x+3)/x$ leaves 3 for every nonzero x. x is a term of the numerator, not a factor, so it cannot be cancelled: at $x=1$, $(1+3)/1=4$, not 3.

Applying it

- Set up the equation from units and the meaning of the unknown.
- A negative or fractional solution may be algebraically correct but impossible for a count.

In your course

- Pearson Edexcel International A Level Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Identity**: An equality that holds for every allowed value of its variable.
- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Watch for: cancelling a term is not the same as cancelling a factor.

P4 · Binomial expansion and valid approximations

How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** 二项式系数: The number of ways to choose a specified number of objects from a set.

The method

- For positive integer n, (a+b)^n is a finite binomial expansion.
- For noninteger n, expand (1+x)^n as 1+nx+n(n-1)x^2/2+... with |x|<1.
- Factor out constants before using this form.

A worked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$$

■ **Worked example**

(1+2x)^5=1+10x+40x^2+80x^3+80x^4+32x^5. For (1+x)^(-1), the first three terms are 1-x+x^2; at x=0.1 this gives 0.91 versus the exact 1/1.1≈0.909091.

exercises.lol exercises.com.cn

$(1 + 2x)^5$: Pascal row × powers of 2

Pascal row 5	1	5	10	10	5	1
$\times 2^k$	1	2	4	8	16	32
coefficient	**1**	**10**	**40**	**80**	**80**	**32**
	1	x	x^2	x^3	x^4	x^5

Where it goes wrong

- An expansion in 2x requires $|2x|<1$ for the infinite series, not merely $|x|<1$.
- Positive-integer expansions are finite and do not have that convergence restriction.

> ■ **Watch out**
>
> **Not true:** Every binomial expansion is a finite polynomial, including powers that are not positive integers. For a negative or fractional power the expansion is an infinite series, valid only for $|x|<1$.

Applying it

- State the range of validity alongside an approximation.
- Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x.

In your course

- Pearson Edexcel International A Level Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Binomial coefficient**: The number of ways to choose a specified number of objects from a set.
- For positive integer n, (a+b)^n is a finite binomial expansion.
- Watch for: an expansion in 2x requires |2x|<1 for the infinite series, not merely |x|<1.

P4 · Conics, parametric curves and tangent reasoning

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single y=f(x).
- This lesson studies **parametric equation** 参数方程: A representation in which coordinates are expressed using a common parameter.

The method

- For x=a cos t,y=b sin t, eliminating t gives $x^2/a^2+y^2/b^2=1$.
- For parametric curves, dy/dx=(dy/dt)/(dx/dt), where dx/dt≠0.
- A zero denominator may signal a vertical tangent.

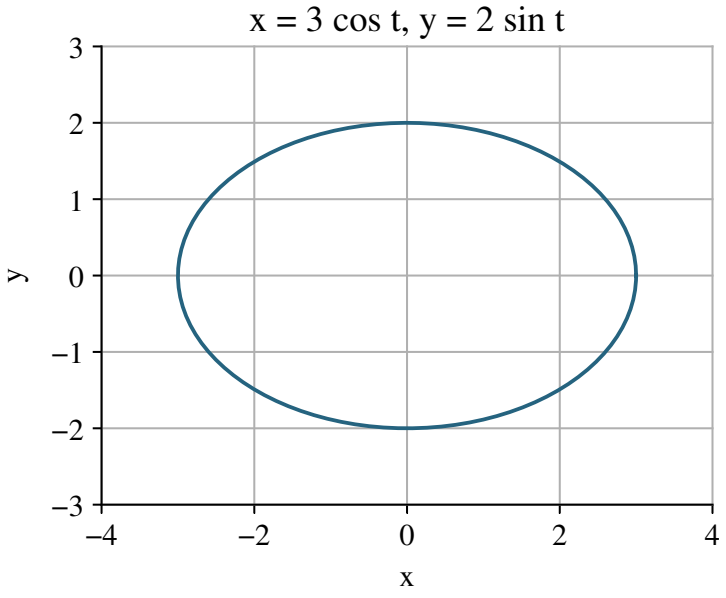
A worked case

$$x = a\cos t, \qquad y = b\sin t, \qquad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

■ **Worked example**

For x=3cos t,y=2sin t at t=π/4, the gradient is (2cos t)/(-3sin t)=-2/3. The point is (3/√2,√2). The area inside the ellipse is πab=6π; its semiaxes are 3 and 2.

exercises.lol exercises.com.cn



Where it goes wrong

- Eliminating a parameter may lose a domain restriction or direction of travel.
- A vertical tangent cannot be assigned a finite dy/dx.
- Distinguish semiaxes from full widths.

> ■ **Watch out**
>
> **Not true:** Eliminating a parameter preserves every restriction and direction automatically. Elimination can lose restrictions: $x=t^2$, $y=t$ gives $x=y^2$, but $x\geq 0$ and the direction of motion must be stated separately.

Applying it

- For advanced coordinate geometry, use the defining equation and a consistent parameter.
- For IAL P4 parametric integration, use the specification's restricted requirements rather than importing all Further Pure conics.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Parametric equation**: A representation in which coordinates are expressed using a common parameter.
- For x=a cos t,y=b sin t, eliminating t gives $x^2/a^2+y^2/b^2=1$.
- Watch for: eliminating a parameter may lose a domain restriction or direction of travel.

P4 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
- This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.

The method

- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- For uv, differentiate to u prime v+uv prime.
- For u/v, use (u prime v-uv prime)/v^2, where $v\neq0$.

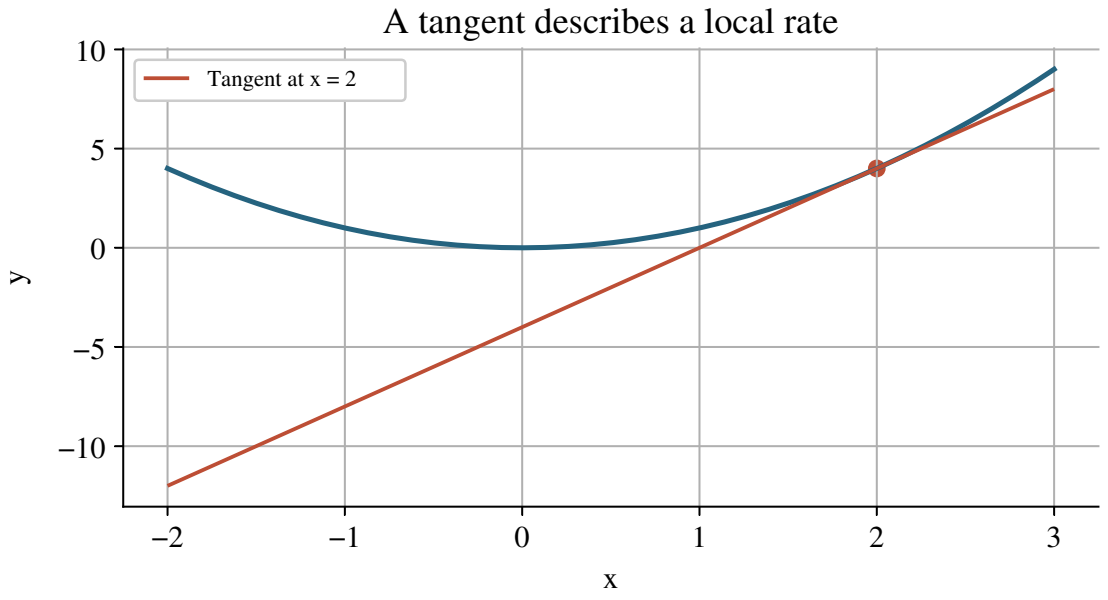
A worked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

Worked example

For y=(3x+1)^4, y prime=4(3x+1)^3×3=12(3x+1)^3. At x=0, the gradient is 12. For $x^2+y^2=25$, differentiate implicitly: 2x+2y y prime=0, so y prime=-x/y when $y\neq0$.

exercises.lol exercises.com.cn



Where it goes wrong

- A derivative of a product is not the product of the derivatives.
- In implicit differentiation, every differentiated function of y brings a dy/dx factor.
- A quotient's denominator is squared.

> ■ **Watch out**
>
> **Not true:** The derivative of u(x)v(x) is u prime times v prime. The product rule is $(uv)'=u'v+uv'$: for $x \cdot x$ the derivative is 2x, not $1\times1=1$.

Applying it

- Choose a useful form before differentiating: expanding a short polynomial may be simpler.
- For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.

In your course

- Pearson Edexcel International A Level Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Chain rule**: The rule that multiplies the outer derivative by the inner derivative for a composite function.
- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- Watch for: A derivative of a product is not the product of the derivatives.

P4 · Substitution, parts and partial fractions

Can we integrate the two factors separately?

- A simple-looking product such as xe^x cannot be integrated by separately integrating each factor.
- This lesson studies **integration by parts** 分部积分法: An integration method based on the derivative of a product.

The method

- Use substitution when an inner derivative appears as a factor.
- By parts, integral u v prime =uv-integral u prime v.
- For a rational function, divide first if needed, then decompose into partial fractions.

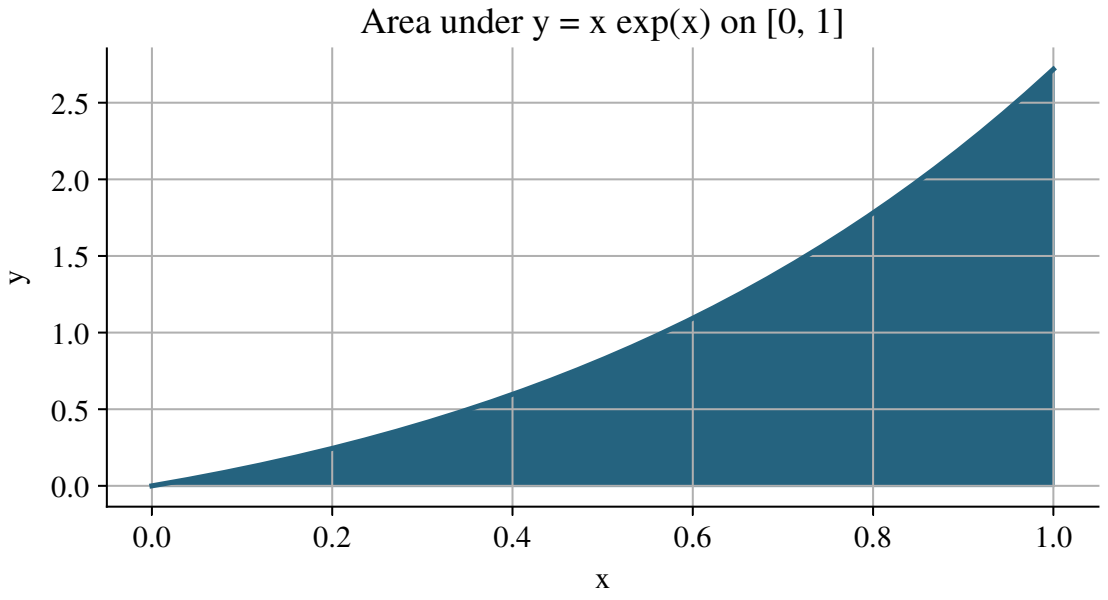
A worked case

$$\int uv'\,dx = uv - \int u'v\,dx$$

■ **Worked example**

For integral xe^x dx, take u=x and v prime=e^x. Then the integral is xe^x-e^x+C. For integral $2x/(x^2+1)$ dx, substitute $w=x^2+1$, dw=2x dx; the answer is $\ln(x^2+1)+C$.

exercises.lol exercises.com.cn



Where it goes wrong

- Choosing u and v prime well matters: the remaining integral should become simpler.
- In a definite substitution, either change the limits or return to x before applying the original limits.

> ■ **Watch out**
>
> **Not true:** The integral of a product equals the product of its separate integrals. A product needs a method such as integration by parts or substitution: $\int x \cdot x\,dx = x^3/3$, not $(x^2/2)(x^2/2)$.

Applying it

- Check by differentiating.
- For volumes of revolution around the x-axis use $V = \pi \int y^2\,dx$; do not confuse the square of a function with the integral of the function.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Integration by parts**: An integration method based on the derivative of a product.
- Use substitution when an inner derivative appears as a factor.
- Watch for: choosing u and v prime well matters: the remaining integral should become simpler.

P4 · Differential equations and numerical solutions

What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
- This lesson studies **initial condition** 初始条件: A specified value that selects a particular solution from a family.

The method

- Separate the variables in dy/dx=ky, integrate both sides and apply an initial condition.
- Include any equilibrium solution excluded when dividing by y.

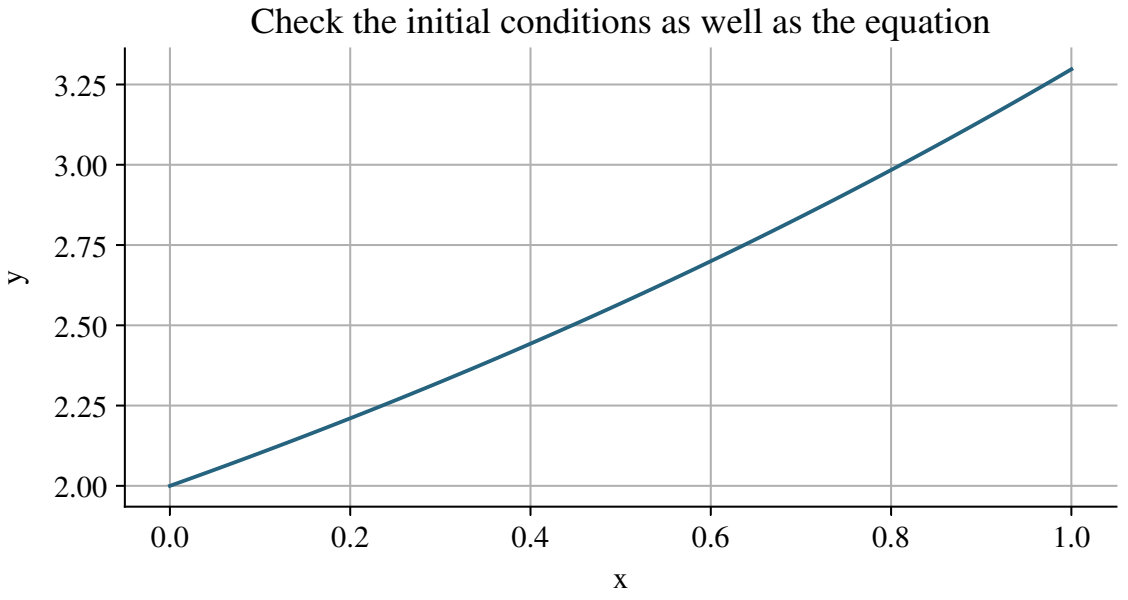
A worked case

$$\frac{dy}{dx} = ky, \qquad y = Ae^{kx}$$

Worked example

For dy/dx=0.5y and y(0)=2, separation gives ln|y|=0.5x+C, hence y=Ae^(0.5x). The initial value gives A=2. Differentiate the result to check the equation and substitute x=0 to check the initial condition.

exercises.lol exercises.com.cn



Where it goes wrong

- An initial condition determines the integration constant; it is not a replacement for integrating.
- Dividing by y can omit an equilibrium solution y=0.
- Euler accuracy depends on step size and the equation.

> ■ **Watch out**
>
> **Not true:** An initial condition never changes the constant in a differential-equation solution. Euler's method depends on the step size; smaller steps usually give a more accurate value.

Applying it

- This unit uses first-order separation and initial conditions.
- Euler numerical integration and second-order complementary functions are excluded from this lesson.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Initial condition**: A specified value that selects a particular solution from a family.
- Separate the variables in dy/dx=ky, integrate both sides and apply an initial condition.
- Watch for: an initial condition determines the integration constant; it is not a replacement for integrating.

P4 · Spatial vectors, lines and angles

Can nonparallel paths still miss each other?

- Two paths in space can be neither parallel nor intersecting. A two-dimensional sketch may conceal their separation.
- This lesson studies **direction vector** 方向向量: A nonzero vector giving the direction of a line.

The method

- A line has r=a+λb with position vector a and nonzero direction b.
- To find an intersection, equate all components and solve for both line parameters.
- For an angle use a · b=|a||b|cosθ where scalar products are in the named syllabus.

A worked case

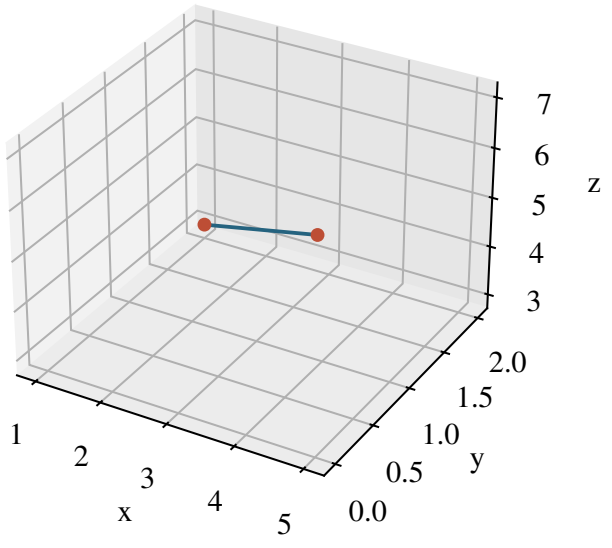
$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}, \qquad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

■ Worked example

For a=(1,2,3),b=(2,-1,2), λ=2 gives r=(5,0,7). The magnitude of b is $\sqrt{}$(4+1+4)=3. With c=(1,0,-1), b · c=2-2=0, so b and c are perpendicular.

exercises.lol exercises.com.cn

$$\mathbf{r} = (1, 2, 3) + \lambda(2, -1, 2)$$



Where it goes wrong

- Satisfying two component equations does not guarantee the third.
- Nonparallel lines may be skew.
- A line's direction vector is not its position vector, and a scalar parameter has no coordinate units.

> **Watch out**
>
> **Not true:** Any two nonparallel lines in three dimensions must intersect. In three dimensions nonparallel lines can be skew: they do not meet.

Applying it

- State whether a requested angle is acute, directed or an angle between lines.
- Use the syllabus-specific plane or distance method only when it is actually required for that unit.

In your course

- Pearson Edexcel International A Level Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Direction vector**: A nonzero vector giving the direction of a line.
- A line has r=a+λb with position vector a and nonzero direction b.
- Watch for: satisfying two component equations does not guarantee the third.

M1 · Kinematics and Newton's laws

Which force accelerates the trolley?

- A trolley accelerates while friction opposes its motion. The engine's force alone does not equal mass times acceleration.
- This lesson studies **resultant force** 合力: The vector sum of all external forces acting on the modelled object.

The method

- Choose a positive direction and draw a force diagram.
- Use resultant F=ma.
- For constant acceleration, v=u+at and s=ut+at^2/2.
- A particle model ignores size; a smooth surface ignores friction.

A worked case

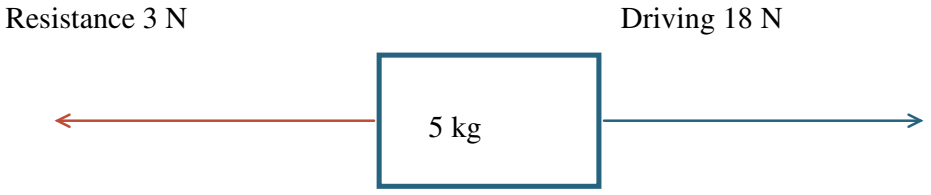
$$F_{\text{resultant}} = ma, \qquad s = ut + \frac{1}{2}at^2$$

■ **Worked example**

A 5 kg trolley has driving force 18 N and resistance 3 N. The resultant is 15 N, so a=15/5=3 m/s^2. Starting from rest for 4 s gives v=0+3×4=12 m/s and s=0×4+3×4^2/2=24 m.

exercises.lol exercises.com.cn

Identify the resultant before using F = ma



Where it goes wrong

- The normal reaction need not equal weight on a slope.
- Connected-particle models need one equation per particle and a consistent acceleration relation.
- The constant-acceleration formulae cannot be used for arbitrary variable acceleration.

■ Watch out

Not true: The constant-acceleration formulae apply unchanged when acceleration varies with time. The suvat formulae need constant acceleration; when acceleration varies, use calculus.

Applying it

- Explain what each idealisation permits and what it leaves out.
- For a velocity-time graph, gradient represents acceleration and signed area represents displacement.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit M1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Resultant force**: The vector sum of all external forces acting on the modelled object.
- Choose a positive direction and draw a force diagram.
- Watch for: the normal reaction need not equal weight on a slope.

M1 · Vectors and transformation geometry

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** 合向量: The vector sum representing the combined displacement or force.

The method

- Add corresponding vector components and subtract position vectors to find a displacement.
- A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.

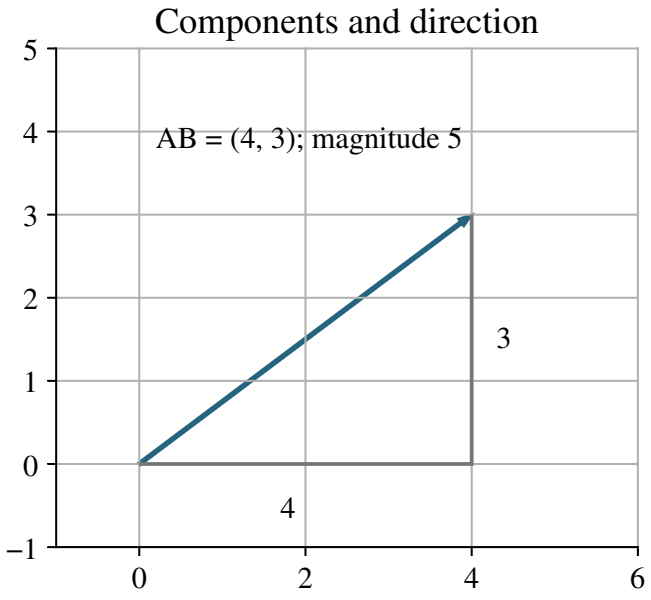
A worked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \qquad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

■ Worked example

With a=(4,1) and b=(1,3), a+b=(5,4). From A=(1,2) to B=(5,5), displacement AB=(4,3). Its magnitude is $\sqrt{(4^2+3^2)}=5$.

exercises.lol exercises.com.cn



Where it goes wrong

- The order of subtraction matters: BA=-AB.
- Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient.
- Negative enlargement reverses position about its centre.

> ■ **Watch out**
>
> **Not true:** AB and BA have the same components. BA goes the opposite way to AB, so BA=−AB: every component changes sign.

Applying it

- Use vector components, scalar multiples and magnitudes for displacements and forces.
- This unit does not require the scalar product.

In your course

- Pearson Edexcel International A Level Mathematics, unit M1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Resultant**: The vector sum representing the combined displacement or force.
- Add corresponding vector components and subtract position vectors to find a displacement.
- Watch for: the order of subtraction matters: BA=-AB.

M1 · Moments, equilibrium and centres of mass

Why does a longer spanner help?

- A long spanner turns a nut more easily. The turning effect depends on perpendicular distance as well as force.
- This lesson studies **moment** 力矩: Force multiplied by the perpendicular distance from its line of action to a pivot.

The method

- For equilibrium, both the resultant force and resultant moment are zero.
- Take moments about a useful pivot to remove unknown forces through that point.
- A combined centre of mass is a mass-weighted mean position.

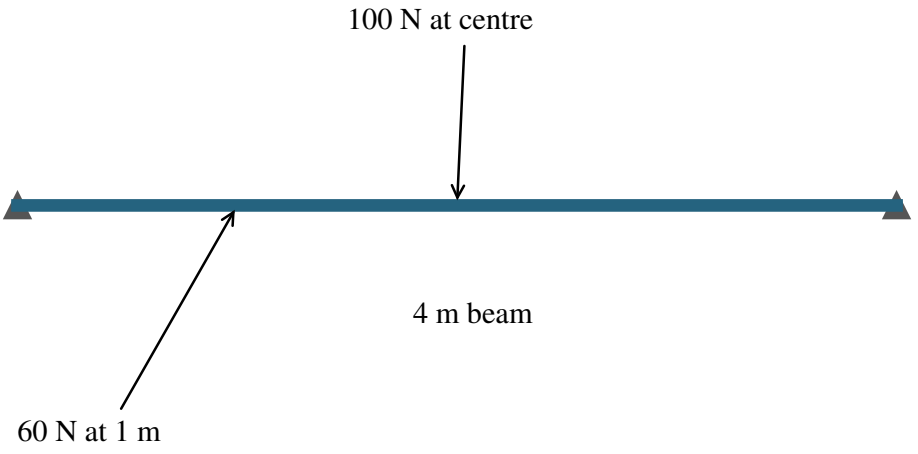
A worked case

$$M = Fd_{\perp}, \qquad \sum M = 0, \qquad \sum \mathbf{F} = \mathbf{0}$$

■ **Worked example**

A uniform 4 m beam weighs 100 N and is supported at both ends. An extra 60 N acts 1 m from the left. Moments about the left give 4R_right=100×2+60×1=260, so R_right=65 N. Vertical equilibrium gives R_left=95 N.

exercises.lol exercises.com.cn



Where it goes wrong

- Use the perpendicular lever arm, not a diagonal distance to the point of application.
- A zero resultant force alone does not prevent rotation.
- A support that would need a negative normal reaction may lose contact.

> ■ **Watch out**
>
> **Not true:** Zero resultant force by itself is enough for a rigid body's equilibrium. Equilibrium also needs the resultant moment to be zero; two equal opposite forces on different lines still turn the body.

Applying it

- For a lamina made from parts, choose a common origin and tabulate each signed mass-area contribution.
- A removed region has negative mass in that calculation, not a new physical negative material.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit M1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Moment**: Force multiplied by the perpendicular distance from its line of action to a pivot.
- For equilibrium, both the resultant force and resultant moment are zero.
- Watch for: use the perpendicular lever arm, not a diagonal distance to the point of application.

M1 · Inclined planes, friction and connected particles

Is friction already at its maximum?

- A crate on a slope is at rest. Friction can adjust to balance the force down the slope; it need not already be at its maximum.
- This lesson studies **limiting friction** 极限摩擦力: The maximum static friction before slipping, equal to μ times the normal reaction in the model.

The method

- Resolve parallel and perpendicular to the plane.
- Weight components are mg $\sin\theta$ and mg $\cos\theta$.
- Static friction satisfies F$\leq\mu$R and equals μR only at the limiting case.
- Write separate equations for connected particles.

A worked case

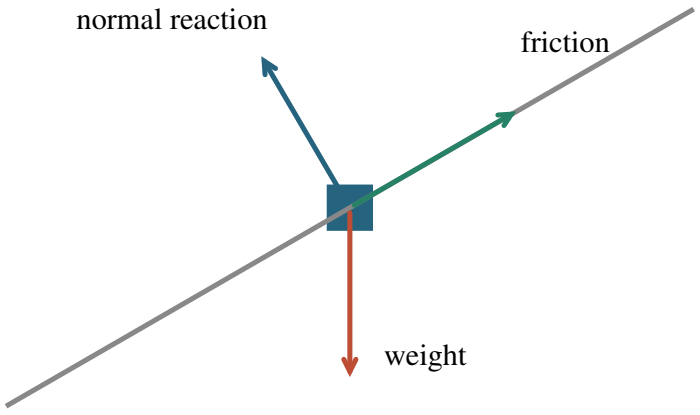
$$R = mg\cos\theta, \qquad F \leq \mu R$$

■ Worked example

A 5 kg crate on a 30° slope has R=5×9.8×cos30°≈42.44 N. The downhill weight component is 24.5 N. With μ=0.6, maximum friction≈25.46 N, so equilibrium is possible with friction 24.5 N.

exercises.lol exercises.com.cn

Resolve weight along and normal to the slope



Where it goes wrong

- Friction opposes motion or the tendency to move, not always the coordinate direction.
- A taut light inextensible string over a smooth pulley gives equal tension and a common acceleration magnitude; each assumption has a job.

> ■ **Watch out**
>
> **Not true:** Static friction is exactly μR even when the body is not about to slip. Static friction is at most μR; it equals μR only when slipping is about to happen.

Applying it

- Start with force diagrams and a proposed direction of motion.
- If the calculated direction conflicts with the friction assumption, revisit the model instead of keeping inconsistent signs.

In your course

- Pearson Edexcel International A Level Mathematics, unit M1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

Key idea

- **Limiting friction**: The maximum static friction before slipping, equal to μ times the normal reaction in the model.
- Resolve parallel and perpendicular to the plane.
- Watch for: friction opposes motion or the tendency to move, not always the coordinate direction.

M2 · Kinematics and Newton's laws

Which force accelerates the trolley?

- A trolley accelerates while friction opposes its motion. The engine's force alone does not equal mass times acceleration.
- This lesson studies **resultant force** 合力: The vector sum of all external forces acting on the modelled object.

The method

- Choose a positive direction and draw a force diagram.
- Use resultant F=ma.
- For constant acceleration, v=u+at and s=ut+at^2/2.
- A particle model ignores size; a smooth surface ignores friction.

A worked case

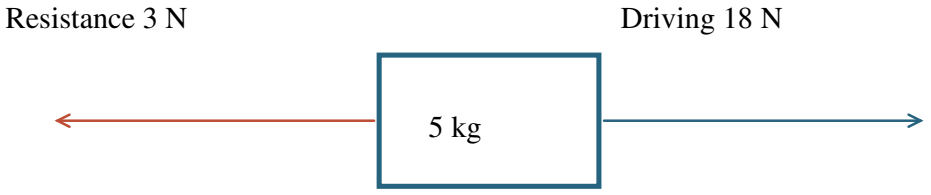
$$F_{\text{resultant}} = ma, \qquad s = ut + \frac{1}{2}at^2$$

Worked example

A 5 kg trolley has driving force 18 N and resistance 3 N. The resultant is 15 N, so a=15/5=3 m/s^2. Starting from rest for 4 s gives v=0+3×4=12 m/s and s=0×4+3×4^2/2=24 m.

exercises.lol exercises.com.cn

Identify the resultant before using F = ma



Where it goes wrong

- The normal reaction need not equal weight on a slope.
- Connected-particle models need one equation per particle and a consistent acceleration relation.
- The constant-acceleration formulae cannot be used for arbitrary variable acceleration.

■ Watch out

Not true: The constant-acceleration formulae apply unchanged when acceleration varies with time. The suvat formulae need constant acceleration; when acceleration varies, use calculus.

Applying it

- Explain what each idealisation permits and what it leaves out.
- For a velocity-time graph, gradient represents acceleration and signed area represents displacement.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit M2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Resultant force**: The vector sum of all external forces acting on the modelled object.
- Choose a positive direction and draw a force diagram.
- Watch for: the normal reaction need not equal weight on a slope.

M2 · Moments, equilibrium and centres of mass

Why does a longer spanner help?

- A long spanner turns a nut more easily. The turning effect depends on perpendicular distance as well as force.
- This lesson studies **moment** 力矩: Force multiplied by the perpendicular distance from its line of action to a pivot.

The method

- For equilibrium, both the resultant force and resultant moment are zero.
- Take moments about a useful pivot to remove unknown forces through that point.
- A combined centre of mass is a mass-weighted mean position.

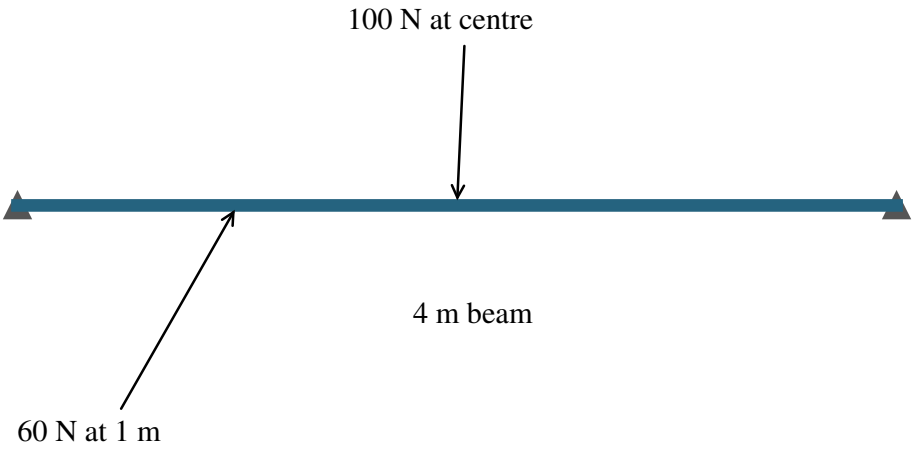
A worked case

$$M = Fd_{\perp}, \qquad \sum M = 0, \qquad \sum \mathbf{F} = \mathbf{0}$$

Worked example

A uniform 4 m beam weighs 100 N and is supported at both ends. An extra 60 N acts 1 m from the left. Moments about the left give 4R_right=100×2+60×1=260, so R_right=65 N. Vertical equilibrium gives R_left=95 N.

exercises.lol exercises.com.cn



Where it goes wrong

- Use the perpendicular lever arm, not a diagonal distance to the point of application.
- A zero resultant force alone does not prevent rotation.
- A support that would need a negative normal reaction may lose contact.

> ■ **Watch out**
>
> **Not true:** Zero resultant force by itself is enough for a rigid body's equilibrium. Equilibrium also needs the resultant moment to be zero; two equal opposite forces on different lines still turn the body.

Applying it

- For a lamina made from parts, choose a common origin and tabulate each signed mass-area contribution.
- A removed region has negative mass in that calculation, not a new physical negative material.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit M2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Moment**: Force multiplied by the perpendicular distance from its line of action to a pivot.
- For equilibrium, both the resultant force and resultant moment are zero.
- Watch for: use the perpendicular lever arm, not a diagonal distance to the point of application.

M2 · Work, energy, power and collisions

What does a slower rebound tell us?

- A ball rebounds slower after hitting a wall. Momentum and kinetic energy answer different parts of the collision story.
- This lesson studies **coefficient of restitution** 恢复系数: The ratio of relative separation speed to relative approach speed along the line of impact.

The method

- Work is force times displacement in its direction.
- Kinetic energy is $mv^2/2$ and gravitational potential energy is mgh.
- Momentum is conserved for an isolated collision; restitution gives the relative speed relation, not conservation of kinetic energy.

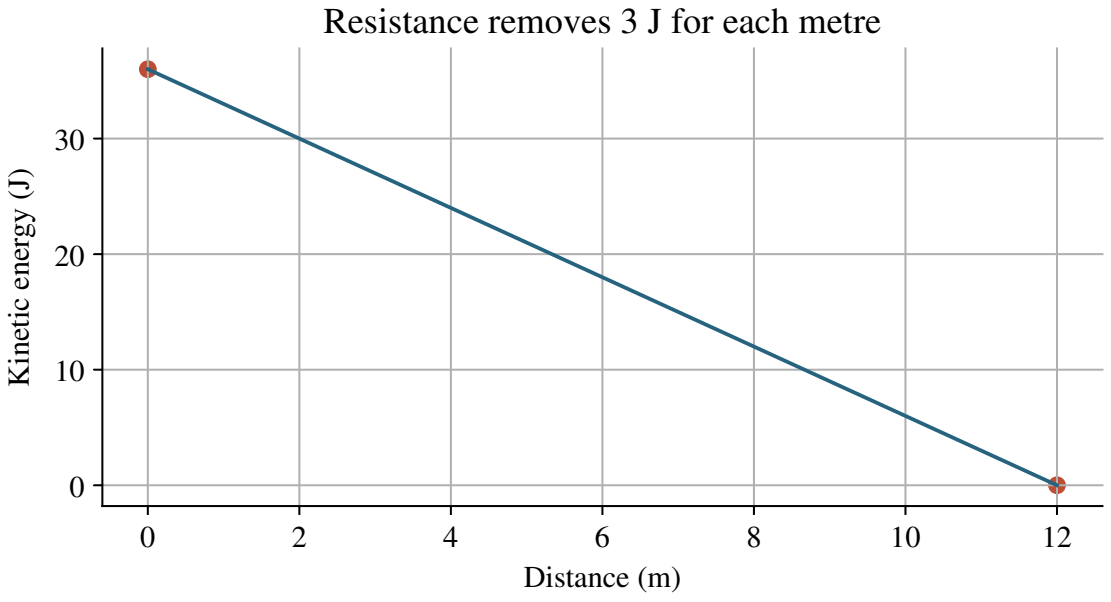
A worked case

$$E_k = \frac{1}{2}mv^2, \qquad W = Fs, \qquad P = \frac{dE}{dt}$$

■ Worked example

A 2 kg particle moving at 6 m/s has kinetic energy $2\times6^2/2=36$ J. If a constant opposing force of 3 N stops it, work=3s=36, giving s=12 m. A wall bounce with approach speed 5 and e=0.6 has rebound speed 3 m/s in the opposite direction.

exercises.lol exercises.com.cn



Where it goes wrong

- Conserve momentum only when the external impulse is negligible.
- Kinetic energy is generally lost in an inelastic collision.
- Power is a rate of energy transfer, not an amount of energy.

■ Watch out

Not true: Every collision conserves kinetic energy. Kinetic energy is conserved only in perfectly elastic collisions; momentum is conserved in every collision.

Applying it

- State the direction signs before writing collision equations.
- For a connected motion problem, choose the whole system for energy when internal tensions cancel; account for frictional work explicitly.

In your course

- Pearson Edexcel International A Level Mathematics, unit M2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Coefficient of restitution**: The ratio of relative separation speed to relative approach speed along the line of impact.
- Work is force times displacement in its direction.
- Watch for: conserve momentum only when the external impulse is negligible.

M2 · Projectile motion and variable acceleration

Why does horizontal motion share vertical time?

- A ball is launched at an angle. Its horizontal and vertical motion share time but follow different equations.
- This lesson studies **projectile** 抛体: A particle moving under gravity after launch in a model that neglects air resistance.

The method

- Resolve initial velocity into horizontal u cosθ and vertical u sinθ.
- With no air resistance, horizontal acceleration is zero and vertical acceleration is -g.
- Use the same time in both components.

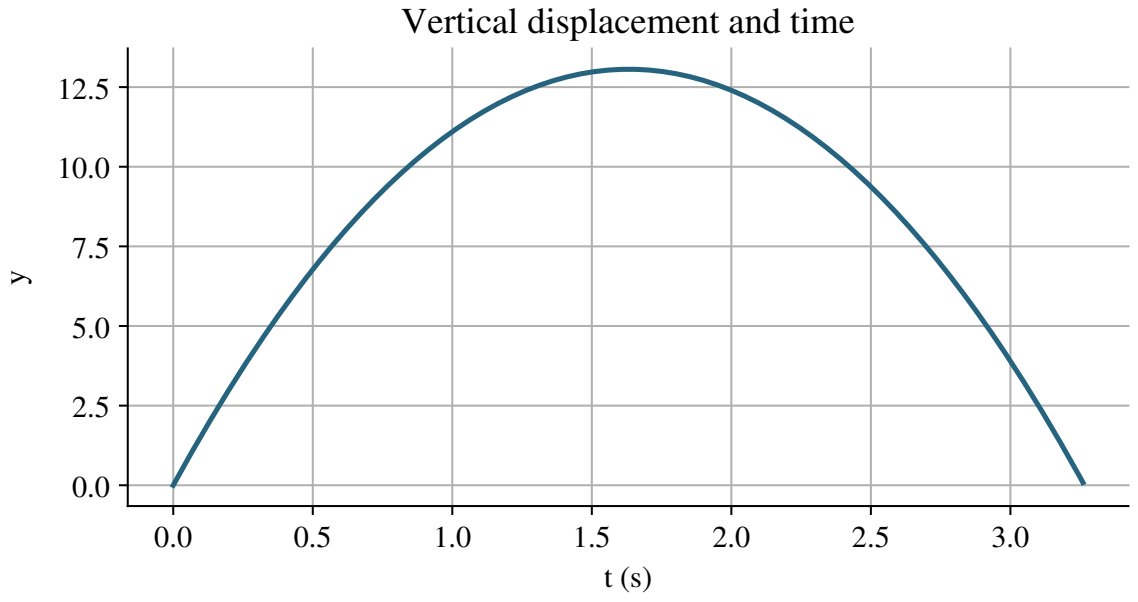
A worked case

$$x = u_x t, \qquad y = u_y t - \frac{1}{2} g t^2$$

■ **Worked example**

For components u_x=12,u_y=16 and g=9.8, at t=2 s the horizontal displacement is x=12×2=24 m and vertical displacement is y=16×2-9.8×2^2/2=12.4 m. At the highest point, v_y=0 but v_x remains 12.

exercises.lol exercises.com.cn



Where it goes wrong

- Zero vertical velocity at the highest point does not mean zero total speed.
- Launch and landing heights need not be equal.
- Do not use a range formula that assumes equal heights without checking them.

> ■ **Watch out**
>
> **Not true:** The total speed of a projectile is zero at its highest point. At the top the vertical velocity is zero, but the horizontal velocity remains, so the speed is not zero.

Applying it

- For variable acceleration, integrate a(t) to get v(t) and use the initial velocity to find the constant, then integrate for displacement.
- Confirm units and the physical time interval.

In your course

- Pearson Edexcel International A Level Mathematics, unit M2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Projectile**: A particle moving under gravity after launch in a model that neglects air resistance.
- Resolve initial velocity into horizontal u cosθ and vertical u sinθ.
- Watch for: zero vertical velocity at the highest point does not mean zero total speed.

S1 · Data summaries, histograms and interpretation

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **frequency density** 频率密度: Frequency divided by class width, used as histogram height.

The method

- Compare an appropriate average and spread in context.
- A histogram uses area for frequency, so height=frequency/class width.
- Grouped estimates assume representative values within intervals.

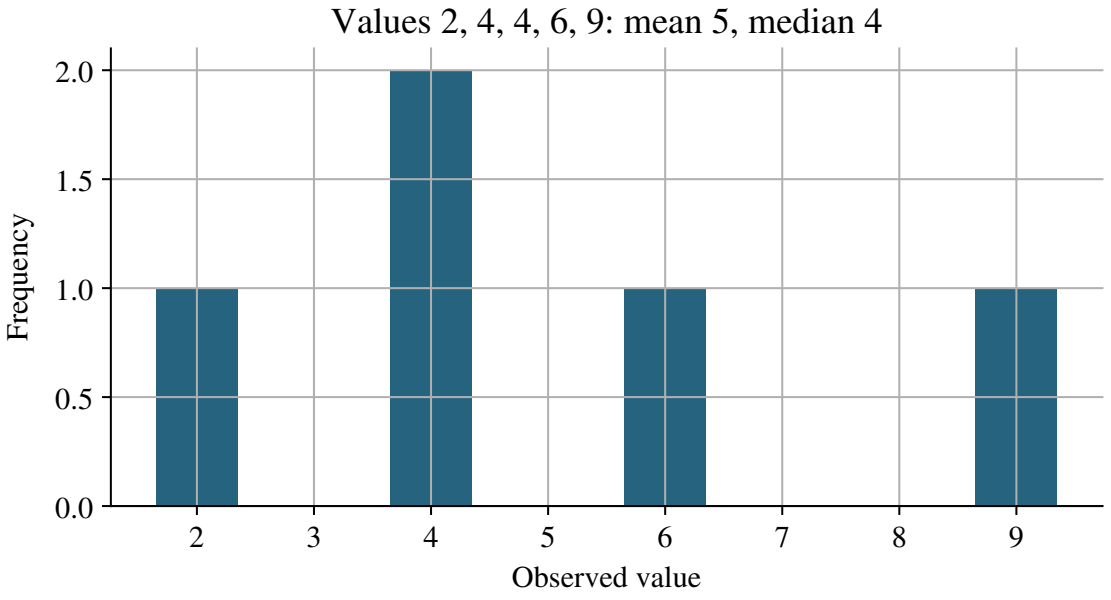
A worked case

$$\overline{x} = \frac{\sum x_i}{n}, \qquad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

■ **Worked example**

A class from 10 to 20 with frequency 30 has density 30/10=3. A class from 20 to 40 with frequency 20 has density 20/20=1. Its wider bar must not be mistaken for a larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.

exercises.lol exercises.com.cn



Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

■ Watch out

Not true: A histogram bar's height equals its frequency, even with unequal class widths. With unequal class widths the height is frequency density; frequency is the area of the bar.

Applying it

- Choose a display that fits the data type.
- Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit S1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Frequency density**: Frequency divided by class width, used as histogram height.
- Compare an appropriate average and spread in context.
- Watch for: the tallest histogram bar need not contain the most observations.

S1 · Probability, trees and conditional reasoning

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** 条件概率: The probability of an event after restricting the sample space to a stated condition.

The method

- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is P(A given B)=P(A∩B)/P(B), for P(B)>0.

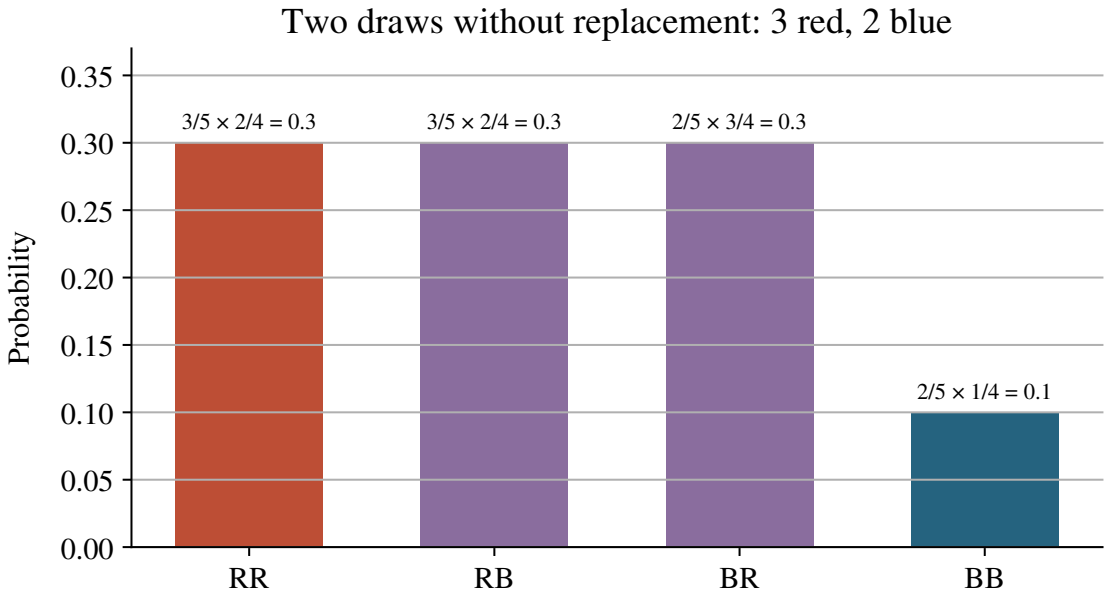
A worked case

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \qquad P(B) > 0$$

■ Worked example

Without replacement, P(two red)=3/5×2/4=3/10. P(one of each)=3/5×2/4+2/5×3/4=3/5. If P(A∩B)=0.12 and P(B)=0.3, P(A given B)=0.4.

exercises.lol exercises.com.cn



Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

> ■ **Watch out**
>
> **Not true:** Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B)=0 \neq P(A)P(B)$.

Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.

In your course

- Pearson Edexcel International A Level Mathematics, unit S1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Conditional probability**: The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

S1 · Regression, financial models and residuals

Is the prediction supported by the data?

- A line predicts bus travel time from distance. A small calculation error matters less than using a model outside its evidence.
- This lesson studies **residual** 残差: The observed value minus the value predicted by a fitted model.

The method

- A linear model y=a+bx has intercept a and slope b with contextual units.
- Inspect residuals and the data range.
- For repeated percentage change use a geometric model; for a loan distinguish principal, rate, repayment and period.

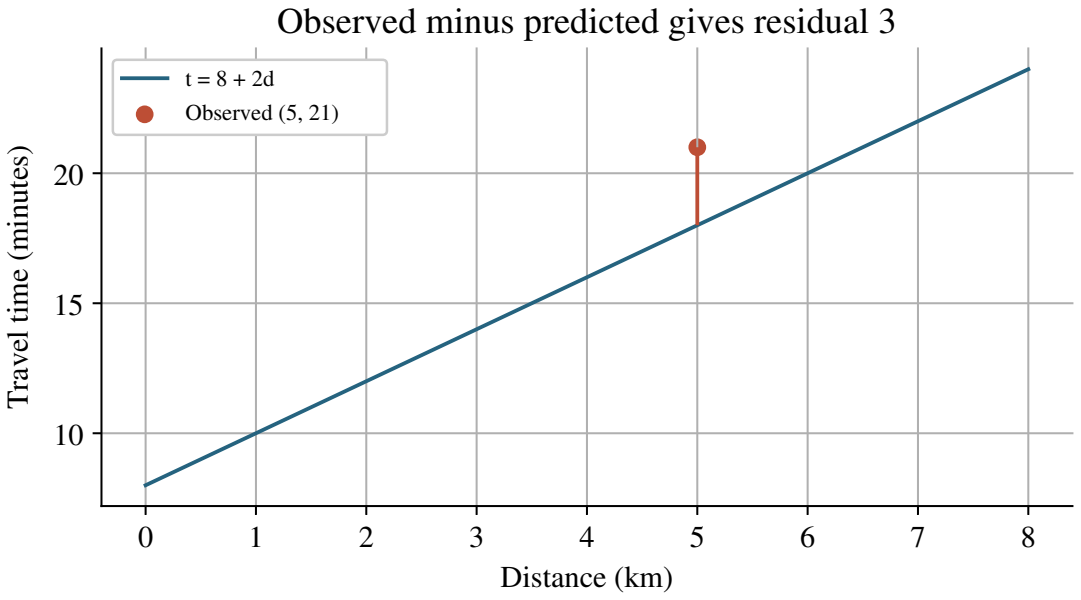
A worked case

$$\widehat{t} = 8 + 2d, \qquad e = t - \widehat{t}$$

■ **Worked example**

The fitted model t=8+2d predicts 18 minutes at d=5 km. If the observed time is 21, the residual is 3 minutes. A ¥1000 deposit at 5% compound annual interest becomes 1000×1.05^3=1157.625 after 3 years.

exercises.lol exercises.com.cn



Where it goes wrong

- A good fit does not prove a causal mechanism.
- A correlation coefficient measures linear association, not the gradient.
- Calculator output must be translated into a model, checked and interpreted.

■ Watch out

Not true: A high correlation permits reliable extrapolation to any distance. A model fitted to data is reliable only near that data; extrapolating far outside it is unreliable.

Applying it

- In IB AI, record the data source, domain, assumptions and calculator method.
- Compare an alternative model and judge predictions against residuals.
- A financial answer must state payment timing and whether interest is compounded.

In your course

- Pearson Edexcel International A Level Mathematics, unit S1.

exercises.lol exercises.com.cn

■ **Key idea**

- **Residual**: The observed value minus the value predicted by a fitted model.
- A linear model y=a+bx has intercept a and slope b with contextual units.
- Watch for: A good fit does not prove a causal mechanism.

S1 · Binomial, normal and Poisson models

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** 期望值: The probability-weighted mean of a random variable.

The method

- For a discrete random variable, sum the probabilities to 1 and calculate E(X)=sum xP(X=x).
- For the normal model use z=(x-μ)/σ and calculate an appropriate area.

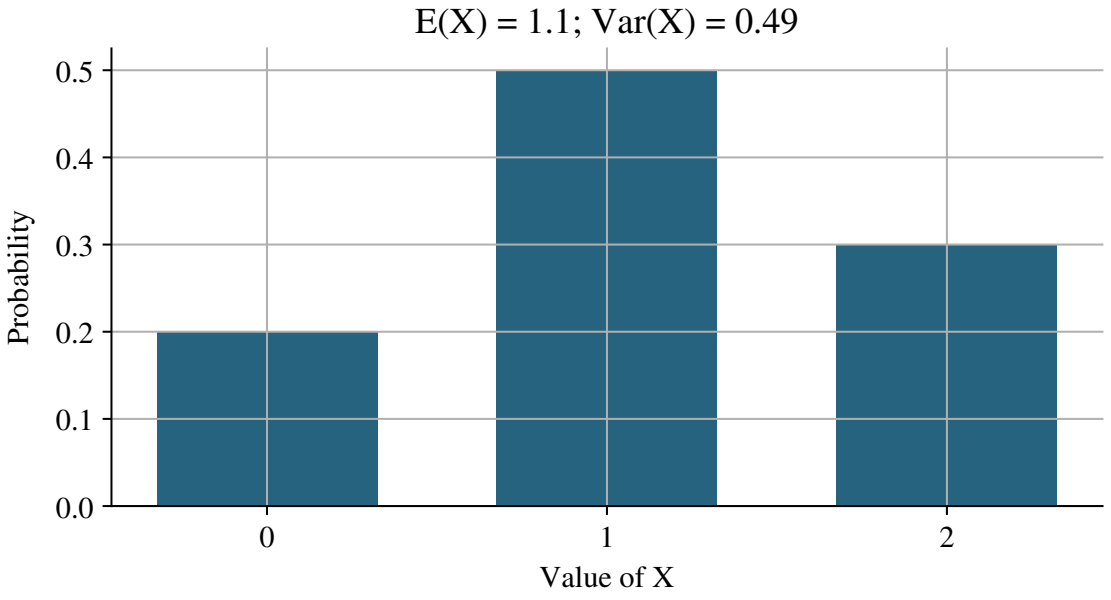
A worked case

$$E(X) = \sum xP(X = x), \quad z = \frac{x - \mu}{\sigma}$$

■ **Worked example**

If X takes 0,1,2 with probabilities 0.2,0.5,0.3, E(X)=0×0.2+1×0.5+2×0.3=1.1. Also $E(X^2)$=0+0.5+1.2=1.7, so Var(X)=1.7-1.1^2=0.49. For μ=100,σ=15, a value of 130 has z=2.

exercises.lol exercises.com.cn



Where it goes wrong

- Not every count is binomial: changing p or dependence can invalidate it.
- For a continuous variable, the probability of one exact value is zero.
- Continuity correction matters when approximating a discrete distribution by a normal one.

Watch out

Not true: A random variable with three possible values must be binomial. The binomial model needs independent trials with a fixed probability; dependent trials break it.

Applying it

- S1 includes discrete random variables and the normal distribution.
- Named binomial and Poisson models are S2 content.

In your course

- Pearson Edexcel International A Level Mathematics, unit S1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Expected value**: The probability-weighted mean of a random variable.
- For a discrete random variable, sum the probabilities to 1 and calculate E(X)=sum xP(X=x).
- Watch for: not every count is binomial: changing p or dependence can invalidate it.

S2 · Binomial, normal and Poisson models

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** 期望值: The probability-weighted mean of a random variable.

The method

- A binomial model needs fixed n, independent trials, two outcomes and constant p.
- E(X)=np and Var(X)=np(1-p).
- For a normal model use z=(x-μ)/σ.
- A Poisson model describes counts with a constant rate and appropriate independence assumptions.

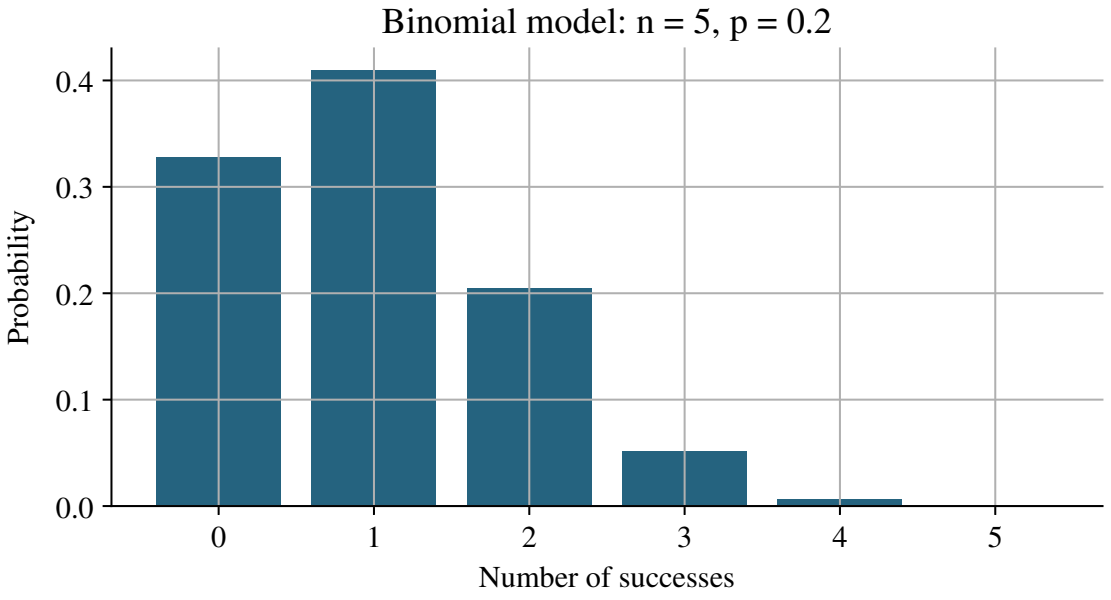
A worked case

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad X \sim B(n, p)$$

■ **Worked example**

For X binomial(5,0.2), P(X=0)=0.8^5=0.32768, E(X)=1 and Var(X)=0.8. For a normal quantity with μ=100,σ=15, the value 130 has z=2. A Poisson mean of 3 per hour gives mean 6 over two hours.

exercises.lol exercises.com.cn



Where it goes wrong

- Not every count is binomial: changing p or dependence can invalidate it.
- For a continuous variable, the probability of one exact value is zero.
- Continuity correction matters when approximating a discrete distribution by a normal one.

> ■ **Watch out**
>
> **Not true:** Every count of successes has a binomial distribution regardless of dependence. The binomial model needs independent trials with a fixed probability; dependent trials break it.

Applying it

- Write the event as an inequality before using calculator distribution functions.
- Distinguish $P(X<k)$, $P(X\leq k)$ and a tail complement.
- State assumptions in context.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit S2.

■ Key idea

- **Expected value**: The probability-weighted mean of a random variable.
- A binomial model needs fixed n, independent trials, two outcomes and constant p.
- Watch for: not every count is binomial: changing p or dependence can invalidate it.

S2 · Sampling and a large data set

Who is missing from the data?

- A weather database has a missing entry and several stations. Treating each row as identical can distort a comparison.
- This lesson studies **sampling frame** 抽样框: The list or population definition from which a sample is selected.

The method

- Identify the population, sampling unit and frame.
- Distinguish random, systematic, stratified, quota and opportunity sampling.
- Missing data is not zero; verify units, dates and variable definitions before comparing samples.

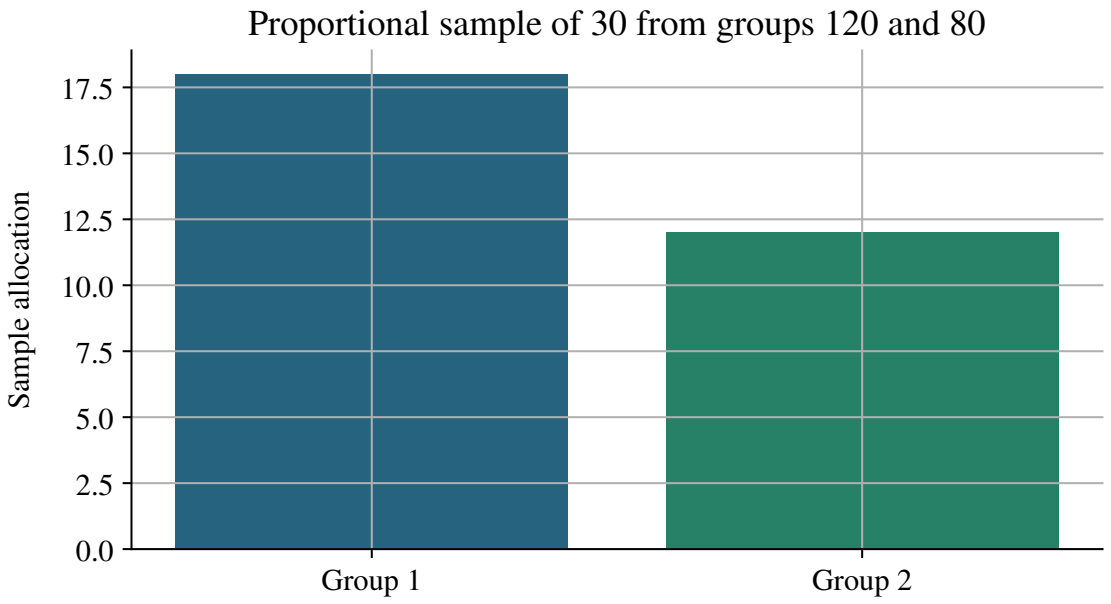
A worked case

$$n_i = n\frac{N_i}{N}$$

■ Worked example

A population has 120 students in one group and 80 in another. A proportional stratified sample of 30 needs $30\times120/200=18$ from the first and 12 from the second. Random selection is then needed within each group.

exercises.lol exercises.com.cn



Where it goes wrong

- A large biased sample remains biased.
- Stratification is not the same as selecting whoever is available from each group.
- AQA large-data-set familiarity requires the actual supplied data and metadata, not invented weather values.

Watch out

Not true: Doubling a biased sample removes its selection bias. A bigger sample collected the same biased way keeps the same bias; only better sampling removes it.

Applying it

- For AQA, use the official large data set in a supervised spreadsheet task: identify a variable, justify a comparison, inspect missing values, create a display and explain a limitation.
- Save the decisions with the analysis.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit S2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Sampling frame**: The list or population definition from which a sample is selected.
- Identify the population, sampling unit and frame.
- Watch for: A large biased sample remains biased.

S2 · Hypothesis testing and contextual conclusions

Could chance explain the result?

- A factory claims that only 10% of items are defective. A sample contains more defects, but chance alone may explain some difference.
- This lesson studies **significance level** 显著性水平: The chosen probability threshold for rejecting a null hypothesis.

The method

- State H_0 and H_1 in population parameters before inspecting the outcome.
- Calculate the appropriate tail probability under H_0.
- Reject H_0 when the evidence meets the specified significance rule; otherwise say there is insufficient evidence.

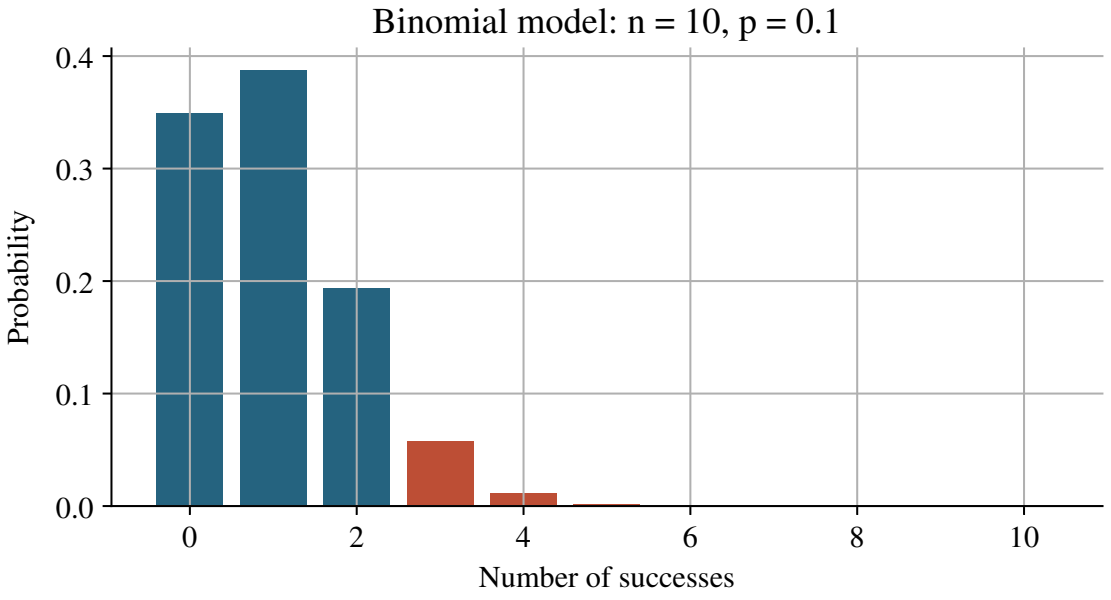
A worked case

$$H_0 : p = p_0, \qquad H_1 : p > p_0$$

■ Worked example

For H_0:p=0.1 and H_1:p>0.1 with n=10, observing at least 3 defects has probability 1-(0.9^10+10×0.1×0.9^9+45×0.1^2×0.9^8)≈0.070191. At 5%, this is insufficient evidence that the defect rate exceeds 10%.

exercises.lol exercises.com.cn



Where it goes wrong

- Failing to reject H_0 is not proof that H_0 is true.
- Choose the tail from H_1, not from whichever tail gives a small result.
- Statistical significance does not measure the practical size of an effect.

> ■ **Watch out**
>
> **Not true:** Failing to reject a null hypothesis proves it is true. Not rejecting the null hypothesis only means there is not enough evidence against it.

Applying it

- Finish with a sentence about the population and the original claim.
- State the model's assumptions and consider whether the sampling procedure supports them.

In your course

- Pearson Edexcel International A Level Mathematics, unit S2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Significance level**: The chosen probability threshold for rejecting a null hypothesis.
- State H_0 and H_1 in population parameters before inspecting the outcome.
- Watch for: failing to reject H_0 is not proof that H_0 is true.

S2 · Continuous distributions and density functions

Is a height the same as a probability?

- A waiting time can take any value in an interval. Its probability comes from area, not the graph's height at one instant.
- This lesson studies **probability density** 概率密度: A nonnegative function whose integral over an interval gives its probability.

The method

- A density f must be nonnegative and integrate to 1 over its support.
- The cumulative distribution F(x) is the integral up to x.
- For a continuous variable P(X=x)=0 and E(X) is the integral of xf(x).

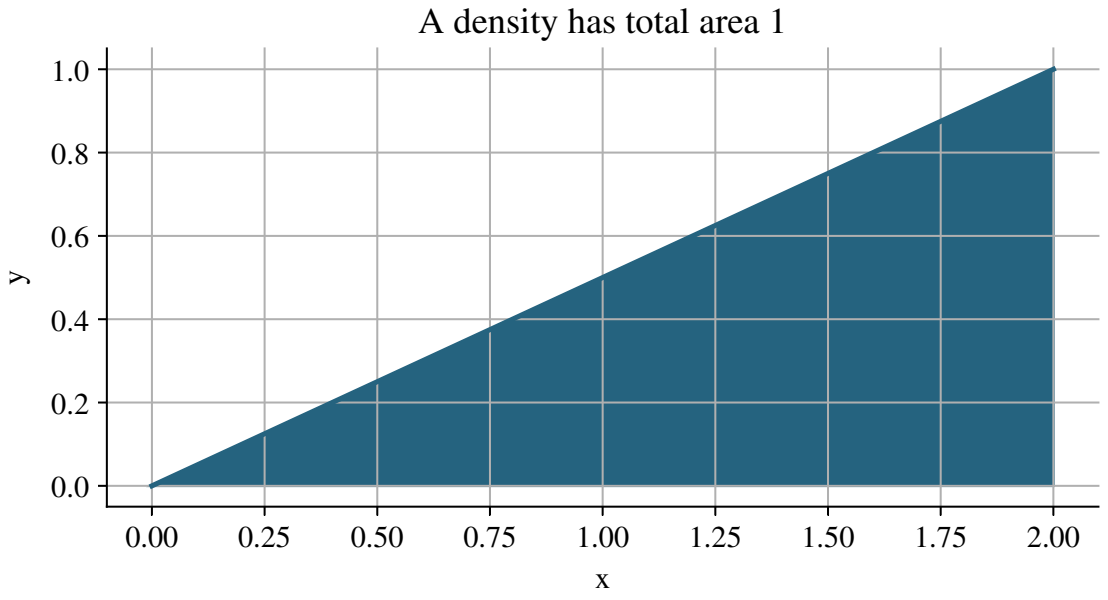
A worked case

$$\int f(x)\,dx = 1, \qquad E(X) = \int x f(x)\,dx$$

■ **Worked example**

Let f(x)=kx on 0≤x≤2 and zero elsewhere. Normalisation gives integral_0^2 kx dx=2k=1, so k=1/2. Then P(X≤1)=integral_0^1 x/2 dx=1/4 and E(X)=integral_0^2 $x^2/2$ dx=4/3.

exercises.lol exercises.com.cn



Where it goes wrong

- Density can exceed 1 without being invalid; total area must equal 1.
- A cumulative distribution cannot decrease.
- Include the zero-density region outside the support when defining a complete model.

> ■ **Watch out**
>
> **Not true:** A continuous variable has P(X=x) equal to the density height f(x). For a continuous variable P(X=x)=0; probabilities are areas under the density curve.

Applying it

- For a uniform distribution, density is reciprocal interval length.
- Find medians or percentiles using the cumulative probability, and use a support-aware integral for moments.

In your course

- Pearson Edexcel International A Level Mathematics, unit S2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

exercises.lol exercises.com.cn

■ **Key idea**

- **Probability density**: A nonnegative function whose integral over an interval gives its probability.
- A density f must be nonnegative and integrate to 1 over its support.
- Watch for: density can exceed 1 without being invalid; total area must equal 1.

D1 · Algorithms, networks and linear programming

Cheapest network or shortest route?

- A school must connect buildings with cable. The shortest route between two buildings and the cheapest whole network are different problems.
- This lesson studies **minimum spanning tree** 最小生成树: A connected cycle-free network joining every vertex with the smallest possible total edge weight.

The method

- Kruskal selects edges in increasing weight while avoiding cycles.
- Dijkstra updates shortest tentative distances from a start.
- Linear programming optimizes a linear objective over a feasible region; inspect vertices and integer restrictions when required.

A worked case

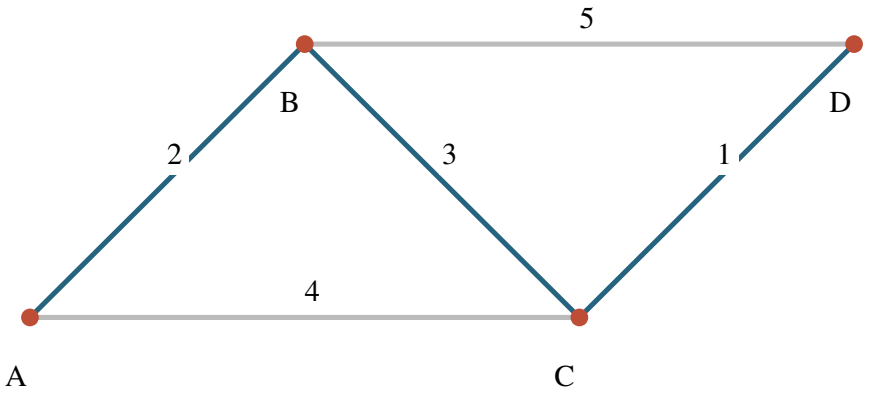
$$Z = 3x + 2y, \qquad x + y \leq 4, \quad x \leq 2, \quad x, y \geq 0$$

■ **Worked example**

For vertices A,B,C,D and edges AB=2,BC=3,AC=4,CD=1,BD=5, Kruskal selects CD,AB,BC for total 6. The shortest A-to-D route is A-B-C-D, also 6, but that equality is incidental. Maximize 3x+2y with x+y≤4,x≤2,x,y≥0: the best vertex is (2,2), value 10.

exercises.lol exercises.com.cn

Minimum spanning tree: weight 6



Where it goes wrong

- A spanning tree has no cycles and joins all vertices.
- The largest single edge is not automatically excluded from every optimal solution.
- A shortest-path algorithm cannot replace a minimum-spanning-tree algorithm.

> ■ **Watch out**
>
> **Not true:** A shortest path between two vertices must also be a minimum spanning tree of the whole network. A shortest path joins two vertices; a minimum spanning tree connects every vertex at least total weight, so they answer different questions.

Applying it

- For critical paths, calculate earliest and latest event times and identify zero-float activities.
- State units and interpret the optimum.
- The existing archive has no D1 pairs, so these original tasks do not establish a reviewed D1 past-paper bank.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit D1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Minimum spanning tree**: A connected cycle-free network joining every vertex with the smallest possible total edge weight.
- Kruskal selects edges in increasing weight while avoiding cycles.
- Watch for: A spanning tree has no cycles and joins all vertices.

D1 · Critical paths and scheduling

Must every activity wait for every other?

- A school event has activities that can run in parallel. Adding every duration overestimates the minimum completion time.
- This lesson studies **float** 浮动时间: The time an activity can be delayed without delaying completion under the network model.

The method

- For an activity network, calculate earliest event times forward and latest event times backward.
- Total float for activity i to j is L_j-E_i-duration.
- A critical activity has zero total float; more than one critical path may exist.

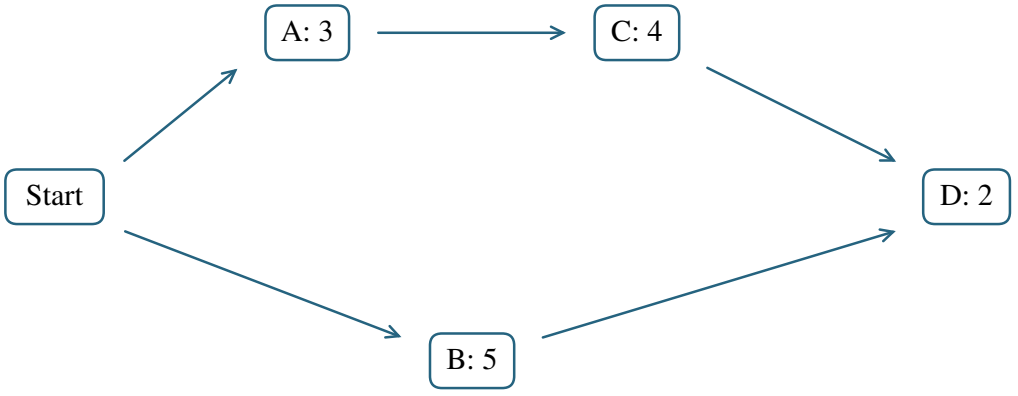
A worked case

$$F_{ij} = L_j - E_i - d_{ij}$$

■ Worked example

Activities A=3 and B=5 start together. C=4 follows A, and D=2 follows both B and C. The earliest completion of C is 7; D must wait until max(5,7)=7 and finishes at 9. Path A-C-D is critical; B has float 2.

Parallel activities meet before D starts



Where it goes wrong

- Do not add durations of independent parallel activities.
- A zero-float activity belongs to a critical path, but there may be several such paths.
- Resource limits can require a schedule longer than the network's theoretical minimum.

> ■ **Watch out**
>
> **Not true:** The minimum project duration is the sum of every activity duration. Activities that run at the same time overlap; the minimum duration is the length of the critical path.

Applying it

- Draw precedence relationships before assigning times.
- Distinguish activity duration from event time.
- Explain what a delay does to the completion date, and record any resource assumptions.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit D1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Float**: The time an activity can be delayed without delaying completion under the network model.
- For an activity network, calculate earliest event times forward and latest event times backward.
- Watch for: do not add durations of independent parallel activities.

D1 · Sorting, packing and network algorithms

Is a fast packing method always optimal?

- A packing method quickly fills boxes, but a fast valid arrangement need not use the smallest number of boxes.
- This lesson studies **algorithm** 算法: A finite set of ordered instructions that solves a defined class of problems.

The method

- Trace the named algorithm exactly, including its tie rules.
- In first-fit packing, place each item in the first available bin that can hold it.
- First-fit decreasing sorts before applying first-fit.
- A heuristic may be valid without being optimal.

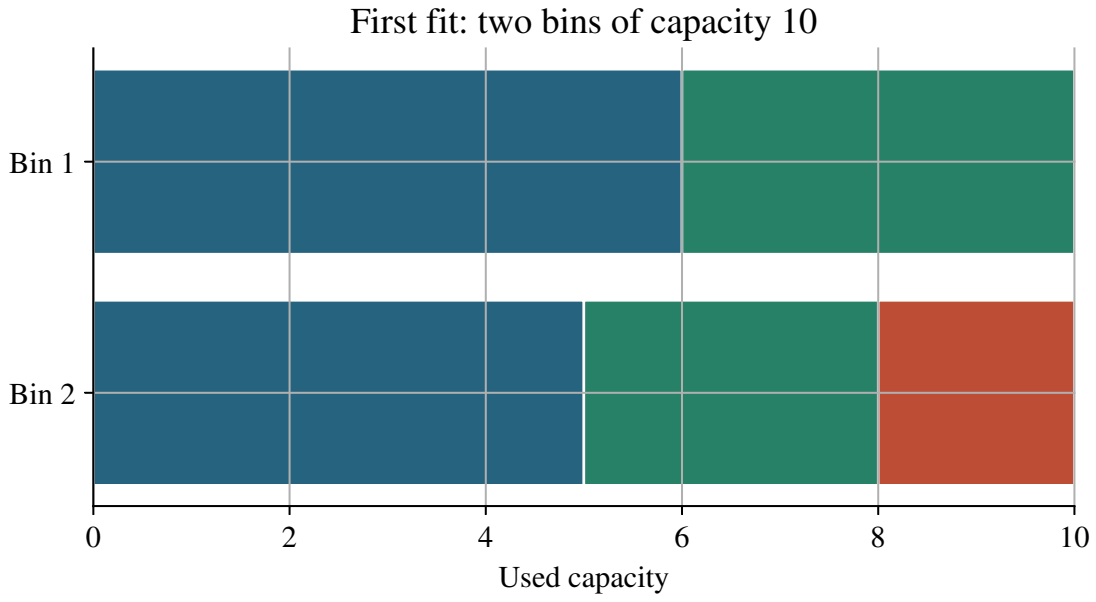
A worked case

$$B_{\min} \geq \left\lceil \frac{\sum s_i}{C} \right\rceil$$

■ Worked example

With bin capacity 10 and items 6,5,4,3,2 in that order, first-fit places 6 and 4 in bin 1, then 5,3,2 in bin 2. It uses 2 bins. The total size is 20, so the lower bound is ceil(20/10)=2; this arrangement is optimal for this instance.

exercises.lol exercises.com.cn



Where it goes wrong

- An example of success does not prove a heuristic is always optimal.
- Keep intermediate lists in a sorting trace; do not jump from input to a sorted final list.
- A shortest-path update must retain predecessor information if a route is requested.

> ■ **Watch out**
>
> **Not true:** A packing heuristic that works well on one example must be optimal. A heuristic gives a good answer quickly but is not guaranteed to be optimal for every input.

Applying it

- For Dijkstra, choose the smallest unsettled tentative label and update its neighbours.
- For route-inspection problems, distinguish a closed route from an open one and identify odd vertices before pairing them.

exercises.lol exercises.com.cn

In your course

- Pearson Edexcel International A Level Mathematics, unit D1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Algorithm**: A finite set of ordered instructions that solves a defined class of problems.
- Trace the named algorithm exactly, including its tie rules.
- Watch for: an example of success does not prove a heuristic is always optimal.

exercises.lol exercises.com.cn