

Mechanics 3

Edexcel IAL M3 (WME03/01); the final mechanics unit of the further-mathematics table

1. Unit M3 at a glance

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WME03/01: 1 hour 30 minutes, 75 marks; the paper is entirely symbolic — exact algebra in m, g, a, L, r . Must-know: Hooke $T = \frac{\lambda e}{L}$, elastic energy $\frac{\lambda e^2}{2L}$, SHM $\ddot{x} = -\omega^2 x$, centripetal $r\omega^2 = \frac{v^2}{r}$, COM $\bar{x} = \frac{\int xy^2 dx}{\int y^2 dx}$.

2. Kinematics with $a = f(x)$

a as a function of x

Replace $\frac{dv}{dt}$ by $v\frac{dv}{dx}$ and integrate in x , closing the constant from a stated instant.

WME03 Q3: $a = 4x - 2$ with $x = 0$, $v = 1$ at $t = 1$ gives $v = 1 - 2x$, then $\frac{dv}{dt} = -2v$ gives $v = e^{2-2t}$.

3. Elastic strings

Elastic strings

$T = \frac{\lambda e}{L}$, energy = $\frac{\lambda e^2}{2L}$; a midpoint string is two half-strings with natural length halved.
 WME03 Q4: energy gain $\frac{1}{2}mga$ at $CD = \frac{3a}{2}$, and with resistance $\frac{1}{5}mg$ the launch
 $\sqrt{3ag/2}$ lands at $V = \sqrt{29ag/10}$. Q1's hinged rod: moments about A give $T = \frac{1}{2}mg$,
 then Hooke gives $L = \frac{24a}{25}$.

4. Variable forces and SHM

SHM

Prove $\ddot{x} = -\omega^2 x$ from the equilibrium tensions first. WME03 Q7: two strings ($\lambda = mg$, $3L$ and $2mg$, $2L$) about E give $\ddot{x} = -\frac{4g}{3L}x$: period $\pi\sqrt{\frac{3L}{g}}$, amplitude $\frac{L}{2}$, $v_{max} = \sqrt{\frac{gL}{3}}$, and the slow time at half speed is a third of a period.

5. Horizontal circles

Horizontal circles

Centreward resolution with $mr\omega^2$ (r is the horizontal radius, not the string) and vertical equilibrium. WME03 Q2: the 30-degree table circle at $\omega = \sqrt{g/2a}$ gives $T = \frac{1}{2}mg$ and the table force $\frac{3}{4}mg$.

6. Vertical circles

Vertical circles

Energy between levels with heights measured from the centre; $R = 0$ (or $T = 0$) at the leaving point; after leaving the horizontal velocity component is conserved. WME03

Q5: $v^2 = 2gr(\cos \alpha - \sin \alpha)$, $\tan \alpha = \frac{2}{3}$ at $R = 0$, then $\cos \theta = \frac{2}{\sqrt{39}}$ at the central level.

7. Centres of mass

Centres of mass

$\bar{x} = \frac{\int_0^a x(a^2 - x^2) dx}{\int_0^a (a^2 - x^2) dx} = \frac{3a}{8}$ for the hemisphere (derivable, not just quotable). WME03 Q6:
 the carved solid has masses 8:1:7, COM $\frac{45a}{112}$ along and $\frac{a}{14}$ off the OC-line, and rests tilted with $\tan \theta = \frac{8}{45}$.

Check yourself

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$a = f(x)$ via $v dv/dx$ with constants closed; Hooke's law and elastic energy for whole and half strings; hinged-rod moments with the tension; SHM proof, period, amplitude, slow-time fractions; two-resolution horizontal circles; vertical-circle energy, $R = 0$, projectile continuation; hemisphere COM derivation; signed-mass composites; tilted equilibrium.