Mechanics 2

Pearson Edexcel · International A-Level · Further Mathematics

Mechanics 2: projectiles, centres of mass, energy and collisions

Mechanics 2 (M2, paper WME02/01, 1 hour 30 minutes, 75 marks, first assessment June 2020) is the IAL applied unit that builds on M1: P1–P4 and M1 are assumed. Take $g = 9.8\ \mathrm{m\,s^{-2}}$ unless the question's model says otherwise (one June 2025 question uses 10). This reference covers variable-acceleration kinematics, centres of mass, work, energy and power, collisions with restitution, and statics of rigid bodies.

Must-know for M2 (not in the booklet):

Family	Formulae
energy	**kinetic energy** $= \frac{1}{2}mv^2$, **potential energy** $= mgh$, work $= Fs$, power $= Fv = \frac{dW}{dt}$
calculus kinematics	$v = \int a\,dt$, $s = \int v\,dt$ (constants closed by initial conditions)
restitution	$e = \dfrac{v_B - v_A}{u_A - u_B}$ along the line, $0 \le e \le 1$
M1 carryover	momentum mv, impulse $= mv - mu$, $F = \mu R$, moments

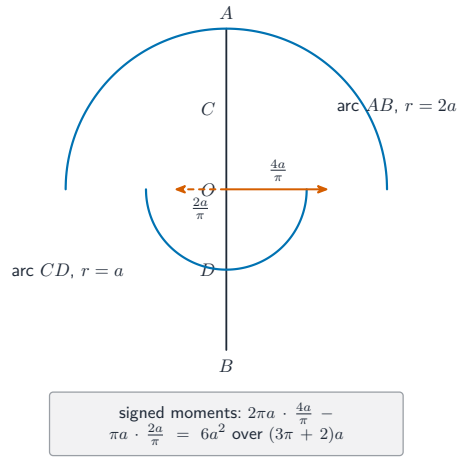
Kinematics with variable acceleration

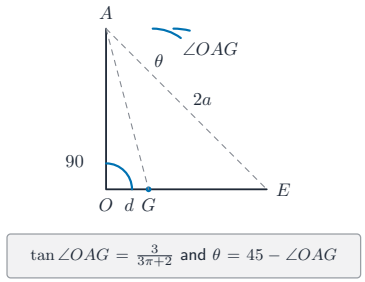
When acceleration is a function of time, **integrate**: $v = \int a\,dt$ and $s = \int v\,dt$, with the constants fixed by the initial conditions ("at rest at O" kills both). For total DISTANCE when the particle turns around, find the stopping time ($v = 0$) and add both legs. Position vectors differentiate component-wise: $\mathbf{v} = \frac{d\mathbf{r}}{dt}$, $\mathbf{a} = \frac{d\mathbf{v}}{dt}$.

Worked example – integration with a trap (WME02 June 2025 Q1). With $a = k(3 - t^2)$ from rest at O and $v(1.5) = 13.5$: $v = k(3t - \frac{t^3}{3})$ (constant zero by rest), and $k(4.5 - 1.125) = 3.375k = 13.5$ gives $k = 4$. Then $s = 6t^2 - \frac{t^4}{3} = \frac{t^2}{3}(18 - t^2)$. For the total distance back to O: $v = \frac{4t}{3}(9 - t^2) = 0$ at $t = 3$, $s(3) = 27$, so the particle travels $27 + 27 = 54$ m – the return leg is a second 27, not a subtraction.

Worked example – vector velocity then impulse (WME02 June 2025 Q2). With $\mathbf{r} = 4\sqrt{t}\,\mathbf{i} - 3t\mathbf{j}$ (metres, t seconds), mass 3 kg: $\mathbf{v} = \frac{d\mathbf{r}}{dt} = 2t^{-1/2}\mathbf{i} - 3\mathbf{j}$, so at $t = 4$ the velocity is $\mathbf{i} - 3\mathbf{j}$. An impulse $(6\mathbf{i} - 3\mathbf{j})$ N s gives $3(\mathbf{v}' - (\mathbf{i} - 3\mathbf{j})) = (6\mathbf{i} - 3\mathbf{j})$, so $\mathbf{v}' = 3\mathbf{i} - 4\mathbf{j}$ and the speed is $5\ \mathrm{m\,s^{-1}}$ – differentiate FIRST, apply the impulse at the stated instant second.

Centres of mass: particles, laminas and wire frameworks





For particles: the **centre of mass** is $\bar{x} = \dfrac{\sum mx}{\sum m}$. For uniform bodies use the table method – split into pieces whose COM you know (booklet: semicircular wire $\frac{2r}{\pi}$, semicircular lamina $\frac{4r}{3\pi}$, sector, arc), weight each piece by MASS (lamina: area; wire: length), and take moments about a convenient line. Pieces on opposite sides of the line carry opposite signs. Composite laminas subtract holes as negative areas.

Worked example – an S-shaped wire framework (WME02 June 2025 Q3). Two semicircular wire arcs, AB of radius $2a$ and CD of radius a, both centred at O on opposite sides of the straight wire ACDB ($AC = DB = a$). Table (about the line AB):

piece	length	COM from AB	moment
arc AB	$2\pi a$	$\frac{2(2a)}{\pi} = \frac{4a}{\pi}$	$8a^2$
arc CD	πa	$-\frac{2a}{\pi}$ (opposite side)	$-2a^2$
straights	$2a$	0	0

Total length $(3\pi + 2)a$; moments $6a^2$; so the centre of mass is $\dfrac{6a}{3\pi + 2}$ from AB. Suspended from A with E on the arc where $\angle AOE = 90°$: the symmetry line OE carries G, so $\tan\angle OAG = \dfrac{3}{3\pi + 2}$, and since $OA = OE$ makes $\angle OAE = 45°$, the angle θ of AE below the vertical satisfies $\theta = 45° - \angle OAG$, giving $\tan\theta = \dfrac{3\pi - 1}{3\pi + 5}$.

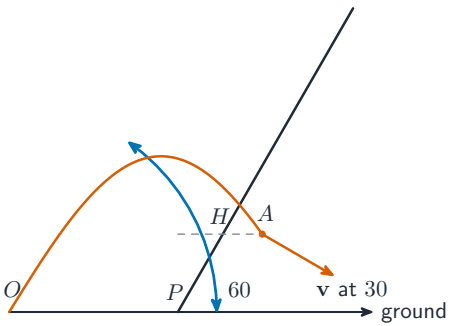
Suspending laminas and frameworks

A freely suspended body hangs with its centre of mass vertically below the point of suspension. The angle asked for is usually between a drawn chord of the body and the vertical: find the angle at the suspension point by right-angled trigonometry with the COM distance, then adjust by any geometry angle (isosceles triangles are the standard source of 45°, 60°).

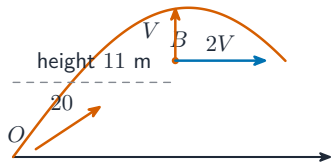
Work, energy and power

Work done by a constant force: Fs along the motion (negative against it); gravity does $\pm mgh$. The **work-energy principle** : total work = change in KE. **conservation of mechanical energy** : energy is conserved when only gravity does work; resistance enters as work $-Rs$. **power** $P = Fv$: at constant speed the driving force balances resistance plus gravity-components, so $v = \frac{P}{F}$; at a given speed the excess force accelerates.

Worked example – a car on a slope (WME02 June 2025 Q4). A 750 kg car climbs a slope with $\sin\alpha = \frac{1}{7}$, resistance 200 N, engine at 20 kW. Constant speed: $\frac{20000}{V} = 750(9.8)\frac{1}{7} + 200 = 1250$, so $V = 16\ \mathrm{m\,s^{-1}}$. At $10\ \mathrm{m\,s^{-1}}$: $2000 - 1250 = 750a$, so $a = 1\ \mathrm{m\,s^{-2}}$. Engine off at A at $10\ \mathrm{m\,s^{-1}}$: work-energy gives $\frac{1}{2}(750)(100) = 750g(d\sin\alpha) + 200d$, i.e. $37500 = 1250d$ and $d = 30$ m – the resistance acts over the SLOPE distance d, gravity over the height $d\sin\alpha$.



perpendicular impact fixes the velocity direction at A



energy: $\frac{1}{2}m(20^2) = \frac{1}{2}m(5V^2) + mg(11)$

Worked example – a projectile closed by energy (WME02 June 2025 Q7). A **projectile** on a parabolic **trajectory** leaves O at $20\ \mathrm{m\,s^{-1}}$ ($g = 10$) is 11 m up when its vertical component is V and its horizontal component $2V$. Energy: $200 = \frac{1}{2}(V^2 + 4V^2) + 110$, so $V = 6$ (components 6 and 12). It meets a plane at 60° to the horizontal PERPENDICULARLY, so the velocity is at 30° below horizontal: speed $= \frac{12}{\cos 30°} = 8\sqrt{3}$, and energy from O gives $200 = \frac{1}{2}(192) + 10H$, so $H = 10.4 \to 10$ m (2 s.f.).

Collisions and restitution

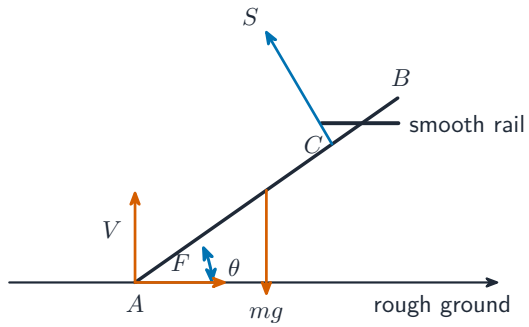
Newton's experimental law, the **coefficient of restitution** : $e = \dfrac{v_B - v_A}{u_A - u_B}$ (separation speed over approach speed, $0 \le e \le 1$; $e = 1$ elastic, $e = 0$ they stick). With conservation of momentum, two equations solve any direct collision. The KE loss always factorises with $(1 - e^2)$: loss $= \frac{m_A m_B}{2(m_A + m_B)}(1 - e^2)(u_A - u_B)^2$ – and the impulse on either body comes from its own momentum change. A rebound from a fixed wall multiplies the speed by e outright.

Worked example – the full collision story (WME02 June 2025 Q5). P (mass $2m$, speed u) strikes Q (mass m) at rest, restitution e. Momentum: $2u = 2v_P + v_Q$; restitution: $eu = v_Q - v_P$. Solving: $v_Q = \frac{2(1+e)u}{3}$, $v_P = \frac{(2-e)u}{3}$. The KE after is $\frac{mu^2}{3}(2 + e^2)$, so the loss is $\frac{mu^2}{3}(1 - e^2)$ – the factor $k = \frac{1}{3}$. If the impulse on Q is $\frac{8}{9}mu$: $m \cdot \frac{2(1+e)u}{3} = \frac{8mu}{9}$ gives $e = \frac{1}{3}$. Q then rebounds from a wall with restitution f to match P's speed $\frac{5}{9}u$: $f \cdot \frac{8}{9}u = \frac{5}{9}u$, so $f = \frac{5}{8}$.

Successive impacts

Three-body and wall chains are bookkeeping: solve the first collision completely, carry the surviving speeds into the next impact, and only equate quantities the question ties together ("same speed", "at rest"). Wall rebounds reverse direction and scale speed by e; keep a signed diagram per stage.

Statics of rigid bodies



moments about A: $S(1.5a) = mga\cos\theta$, and $F = S\sin\theta$ at limiting equilibrium

A **rigid body** in equilibrium needs zero resultant force AND zero resultant **moment** about any point. Choose the pivot to eliminate unknowns (a smooth rail's single unknown reaction is why moments about the floor contact pay off). Resolve horizontally and vertically for the contact components; on a rough floor at **limiting**

equilibrium **friction** $F = \mu R$ with F horizontal.

Worked example – a rod against a smooth rail (WME02 June 2025 Q6). A uniform rod AB (mass m, length $2a$) rests with A on rough ground against a smooth rail at C, $AC = 1.5a$, inclination θ, the **normal reaction** S of the rail, perpendicular to the rod. Moments about A: $S(1.5a) = mg\,a\cos\theta$, so $S = \frac{2mg\cos\theta}{3}$. Vertically: $V = mg - S\cos\theta = \frac{mg}{3}(3 - 2\cos^2\theta)$. Limiting equilibrium with $\mu = \frac{4}{7}$: $F = S\sin\theta$, so $\frac{4}{7} \cdot \frac{mg}{3}(3 - 2\cos^2\theta) = \frac{2mg}{3}\cos\theta\sin\theta$. Dividing by $\cos^2\theta$ and using $\sec^2\theta = 1 + \tan^2\theta$: $12\tan^2\theta - 14\tan\theta + 4 = 0$, i.e. $6\tan^2\theta - 7\tan\theta + 2 = 0$.

Checklist before you call this unit done

- Integrate $a(t)$ to v and s with constants from the initial conditions; handle total distance via the turning point; differentiate $\mathbf{r}(t)$ for vector velocity and apply impulses as $\mathbf{I} = m\Delta\mathbf{v}$.
- Build COM tables (mass/area/length weighting) with signed distances for opposite-side pieces and negative holes; use the booklet's wire and lamina results; find suspension angles by trigonometry at the suspension point plus any isosceles-triangle angle.
- Apply work-energy and conservation of mechanical energy, with resistance work over slope distance and $P = Fv$ both ways.
- Solve direct collisions with momentum + restitution; state both speeds; compute the $(1 - e^2)$ energy loss and per-body impulses; chain wall rebounds and second collisions with a signed diagram.
- For rigid bodies: moments about the unknown-killing pivot, resolve for components, and close limiting-equilibrium problems into an equation in $\tan\theta$.