Mechanics 1

Pearson Edexcel · International A-Level · Further Mathematics

Mechanics 1: models, motion and forces

Mechanics 1 (M1, paper WME01/01, 1 hour 30 minutes, 75 marks, first assessment June 2019) is the applied unit of International AS Mathematics: P1 and P2 are assumed. Take $g = 9.8\ \mathrm{m\,s^{-2}}$ unless told otherwise, and give inexact answers to 2 or 3 significant figures. This reference covers modelling assumptions, vectors in mechanics, constant-acceleration kinematics, Newton's laws with connected particles, momentum and impulse, friction, statics and moments.

Must-know for M1 (not in the formula booklet – memorise):

Family	Formulae
momentum	momentum $= mv$, impulse $= mv - mu$
suvat	$v = u + at$, $s = ut + \frac{1}{2}at^2$, $s = vt - \frac{1}{2}at^2$, $v^2 = u^2 + 2as$, $s = \frac{1}{2}(u+v)t$
forces	resultant $= ma$ (component-wise), weight $= mg$, friction $F = \mu R$ when moving
vectors	$\lvert a\mathbf{i} + b\mathbf{j}\rvert = \sqrt{a^2+b^2}$, bearing from atan in the correct quadrant

Mathematical models in mechanics

The words in a stem are instructions. A **particle** has mass but no size; a **lamina** has area but no thickness; a **rod** has length but no thickness; a **rigid body** keeps its shape. **Light** means massless (a light rope transmits tension unchanged), **uniform** means the centre of mass sits at the midpoint, an **inextensible string** keeps both ends moving with the same speed, and a **smooth surface** versus a **rough surface** decide whether friction acts. A **light smooth pulley** changes a tension's direction, not its size. State the assumption you use – "smooth, so no friction" – because the marks are in the modelling as much as the algebra.

Vectors in mechanics

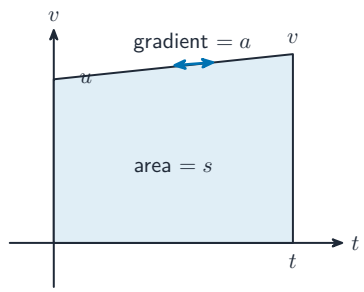
Displacements, velocities, accelerations and forces are vectors: $(3\mathbf{i} - 4\mathbf{j})$ has magnitude 5 and direction $\tan^{-1}(-4/3)$ from the positive x-axis. Resolve into components, work component-wise, and combine at the end. The **resultant** of $\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} + \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ is $\begin{pmatrix} a_1 + a_2 \\ b_1 + b_2 \end{pmatrix}$. With $\mathbf{i}$ east and $\mathbf{j}$ north, a **bearing** is measured clockwise from north: the vector $-5\mathbf{i} - 12\mathbf{j}$ points south-west, at $\tan^{-1}(5/12) = 22.6°$ west of due south, so its bearing is $180 + 23 = 203°$.

Worked example – position vectors and bearings (WME01 June 2025 Q2). Particle A has $\mathbf{r}_A(2) = (-5\mathbf{i}+16\mathbf{j})$ and $\mathbf{r}_A(5) = (10\mathbf{i}+4\mathbf{j})$. Velocity $= \dfrac{\Delta\mathbf{r}}{\Delta t} = \dfrac{15\mathbf{i} - 12\mathbf{j}}{3} = 5\mathbf{i} - 4\mathbf{j}$, so $\mathbf{r}_A(t) = (-5\mathbf{i} + 16\mathbf{j}) + (t-2)(5\mathbf{i} - 4\mathbf{j}) = (5t - 15)\mathbf{i} + (24 - 4t)\mathbf{j}$. With B at $\mathbf{s}_B = -5\mathbf{i} + 7\mathbf{j} + t(2\mathbf{i} - 3\mathbf{j})$, at $t = 5$: $\overrightarrow{AB} = (5\mathbf{i} - 8\mathbf{j}) - (10\mathbf{i} + 4\mathbf{j}) = -5\mathbf{i} - 12\mathbf{j}$, bearing 203°.

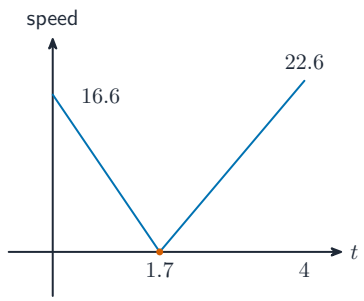
Kinematics of a particle in a straight line

Constant acceleration means the suvat formulae apply once per phase; the discipline is signs. Choose positive, write s, u, v, a, t with signs for each phase, and pick the formula that avoids the unknown you do not want. Graphs: on a velocity–time graph the gradient is acceleration and the area is displacement (area below the axis counts negative); a speed–time graph cannot go negative, so upward motion then downward motion appears as a V.

Worked example – up and down (WME01 June 2025 Q3). A ball is projected vertically up from A, 12 m above the ground at B, and takes 4 s to reach B. Taking up positive with A as origin, $s = -12$: $-12 = 4U - 4.9(16)$, so $U = 16.6\ \mathrm{m\,s^{-1}}$ (2 s.f. 17). Speed at the ground: $v^2 = 16.6^2 + 2(9.8)(12) = 510.8$, so $v = 22.6\ \mathrm{m\,s^{-1}}$. The speed–time graph is a V: falling from 16.6 to zero at the top (after $16.6/9.8 = 1.7$ s), then rising to 22.6 at $t = 4$ – the downward leg is longer and steeper.



velocity–time: gradient accelerates, area displaces



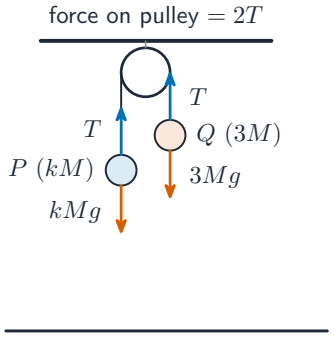
speed–time V: vertex on the axis, downward leg longer

Newton's laws and connected particles

Newton's second law, applied one way along a line: $F_{\text{net}} = ma$ with every force resolved and signed. For two **connected particles** on a light inextensible string, write an equation of motion for EACH body (same tension, same speed magnitude), then eliminate T. On an inclined plane resolve along and perpendicular to the slope; $\tan\theta = 3/4$ means $\sin\theta = 3/5$, $\cos\theta = 4/5$.

Worked example – two ropes on a barge (WME01 June 2025 Q4). Forces of 800 N at 25° above the line of motion and 500 N at α below it pull a barge straight along the canal. Perpendicular to the motion there is no acceleration: $800\sin 25° = 500\sin\alpha$, so $\sin\alpha = 0.676$ and $\alpha = 43°$. Along the motion, with mass 15000 kg and resistance 750 N: $800\cos 25° + 500\cos\alpha - 750 = 15000a$, giving $a = 0.023\ \mathrm{m\,s^{-2}}$ – three-decimal answers are expected when the numbers are this small.

Worked example – pulley, start to finish (WME01 June 2025 Q8). Balls P (mass kM) and Q (mass $3M$, $k < 3$) hang over a fixed light smooth pulley and accelerate at $\frac{g}{5}$. Equations of motion: $T - kMg = kM \cdot \frac{g}{5}$ (P up) and $3Mg - T = 3M \cdot \frac{g}{5}$ (Q down). Eliminating T: $3Mg - kMg = \frac{g}{5}(3M + kM)$, so $5(3-k) = 3+k$ and $k = 2$. The string pulls the pulley down with $2T$: with $M = 0.5$ kg, $T = 1(9.8 + 1.96) = 11.76$ N, so the force is 23.5 N. When Q lands after falling 2.5 m its speed is $\sqrt{2 \cdot \frac{g}{5} \cdot 2.5} = \sqrt{g} = 3.13$; if it rebounds at $0.4\ \mathrm{m\,s^{-1}}$, the impulse on Q is $1.5(3.13 + 0.4) = 5.3$ N s. P then coasts upward against gravity alone for $\frac{\sqrt{g}}{g} = 0.32$ s after a $5/\sqrt{g} = 1.60$ s descent phase: total time to rest ≈ 1.9 s.



P accelerates up and Q down at $\frac{g}{5}$; same string, same tension: $T - kMg = \frac{kMg}{5}$ and $3Mg - T = \frac{3Mg}{5}$

Momentum and impulse

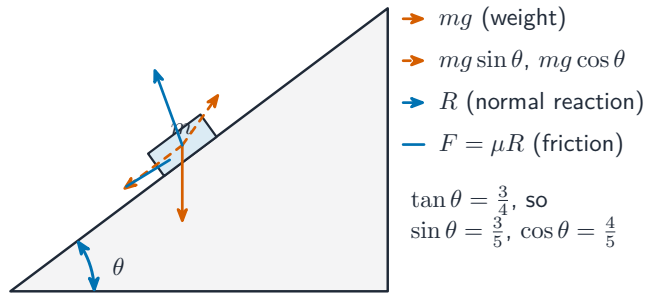
Momentum mv and **impulse** $I = mv - mu$ are signed along the line. Newton's third law makes the impulse exerted on each of two colliding bodies equal in magnitude and opposite in direction – use one of them, not both. For a direct collision, conservation of momentum $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ needs consistent signs; restitution is NOT in M1.

Worked example – impulse on both bodies (WME01 June 2025 Q1). P (mass km, speed $5u$) catches Q (mass $3m$, speed $2u$); afterwards P still moves forward at $2u$ and Q at v. The impulse on Q is $4.5mu$: $3m(v - 2u) = 4.5mu$, so $v = 3.5u$. The impulse on P is equal and opposite: $km(5u - 2u) = 4.5mu$, so $k = 1.5$.

Friction

On a **rough** surface the contact force splits into the **normal reaction** R (perpendicular) and friction F (along, opposing motion or its tendency). Moving: $F = \mu R$ with μ the **coefficient of friction**. In equilibrium $F \le \mu R$, with equality at limiting equilibrium – the phrase "on the point of moving" licenses $F = \mu R$.

Worked example – a package on a rough slope (WME01 June 2025 Q6). With $\tan\theta = \frac{3}{4}$ and $\mu = \frac{1}{4}$: perpendicular, $R = mg\cos\theta = \frac{4}{5}mg$; along (up positive, decelerating): $-mg\sin\theta - \frac{1}{4}R = ma$, so $a = -g(\frac{3}{5} + \frac{1}{5}) = -\frac{4}{5}g$ – the deceleration is $\frac{4}{5}g$, and the answer must stay as an exact fraction of g. Held at rest on the slope by a force X up the plane (mass 2 kg): $X + \frac{1}{4}(2g \cdot \frac{4}{5}) = 2g \cdot \frac{3}{5}$, so $X = \frac{4}{5}g = 7.84$ N.



resolve both ways: $R = \frac{4}{5}mg$ and $a = g(\sin\theta + \mu\cos\theta) = \frac{4}{5}g$ when $\mu = \frac{1}{4}$

Statics of a particle

A particle is in **equilibrium** when the resultant force is zero: resolve in two perpendicular directions and set both totals to zero, or draw a closed vector triangle. Weight mg down, **normal reaction** perpendicular to the surface, **tension** along a string away from the body (thrust along a rod pushes), friction along the surface opposing slipping. On rough ground at rest, F is whatever closes the polygon, capped at μR.

Worked example – forces in equilibrium (original). A particle of weight 50 N hangs from two light strings at 40° and 60° to the vertical. Resolving horizontally: $T_1\sin 40° = T_2\sin 60°$; vertically: $T_1\cos 40° + T_2\cos 60° = 50$. From the first, $T_2 = 0.742T_1$; then $T_1(0.766 + 0.371) = 50$, so $T_1 = 44$ N and $T_2 = 33$ N (2 s.f.).

Moments

The **moment of a force** about a point is force × perpendicular distance, with a sign per sense. A rigid body in equilibrium has zero resultant moment about ANY point – so pick the pivot that kills the most unknowns. "On the point of tilting about X" means the reaction elsewhere has dropped to zero and moments are taken about X.

Worked example – a rod on a ledge (WME01 June 2025 Q5). A non-uniform rod AB of length $3a$ rests on a horizontal ledge with the edge at C, $AC = 2a$, centre of mass d from A. An upward force P at B tilts it about A: moments about A give $3aP = Wd$. A downward force $1.25P$ at B tilts it about C: $1.25Pa = W(2a - d)$. Substituting $P = \frac{Wd}{3a}$ (in units of a): $1.25 \cdot \frac{d}{3} = 2 - d$, so $1.25d = 6a - 3d$, giving $4.25d = 6a$ and $d = \frac{24a}{17}$. The lesson: each tilting case has its own pivot, and only the weight's moment survives unchanged between them.

Checklist before you call this unit done

- Name the modelling assumptions in play (particle, light rod, inextensible string, smooth/rough) and say what each buys you.
- Work with position and velocity vectors component-wise; convert directions to bearings; find relative positions at a common time.
- Solve one-phase and multi-phase suvat problems with signs fixed per phase; read gradients and areas off motion graphs; sketch the speed-time V for up-then-down motion.
- Apply $F = ma$ per body along the line of motion; handle connected particles over pulleys with paired equations; find the force on a pulley as $2T$.
- Use impulse-momentum on ONE body of a collision pair; conserve momentum with consistent signs.
- Split contact forces into R and F; apply $F = \mu R$ when moving or limiting; resolve on inclined planes using $\sin\theta = 3/5$, $\cos\theta = 4/5$ for $\tan\theta = 3/4$.
- Close force polygons for equilibrium in two directions; cap static friction at μR.
- Take moments about the pivot that removes unknowns; switch pivots when the tilting point changes.