

Solutions

2.1 $\sum d^2 = 1 + 0 + 4 + 1 + 1 + 0 + 1 + 0 = 8$: $r_s = 1 - \frac{48}{8(63)} = 1 - 0.0952 = 0.905$ (3 dp).

2.2 $2X - Y \sim N(2(10) - 6, 4(4) + 9) = N(14, 25)$.

2.3 $X - Y \sim N(4, 13)$: $z = \frac{6-4}{\sqrt{13}} = 0.555$, so $P = 1 - \Phi(0.555) = 1 - 0.7103 = 0.2897$.

3.1 (a) The rank differences (Judge 1 – Judge 2) are $-1, 2, 2, 0, 0, 6, -3, -2, -2, -2$, so $\sum d^2 = 1 + 4 + 4 + 0 + 0 + 36 + 9 + 4 + 4 + 4 = 66$ and $r_s = 1 - \frac{6(66)}{10(99)} = 1 - \frac{396}{990} = 0.6$.
(b) $H_0: \rho = 0$, $H_1: \rho > 0$. The 1% one-tailed critical value for $n = 10$ is 0.7455. Since $0.6 < 0.7455$, do not reject H_0 – insufficient evidence of a positive correlation between the two judges' rankings.

3.2 (a) $S_1 + S_2 + S_3 \sim N(54, 3(0.0009)) = N(54, 0.0027)$, standard deviation $\sqrt{0.0027} = 0.05196$. $z = \frac{54.1-54}{0.05196} = 1.924$, so $P = 1 - \Phi(1.924) = 1 - 0.9726 = 0.0274$. (b) $C_1 - C_2 \sim N(0, 2(0.0009)) = N(0, 0.0018)$, standard deviation $\sqrt{0.0018} = 0.04243$. By symmetry, $P(|C_1 - C_2| > 0.02) = 2(1 - \Phi(\frac{0.02}{0.04243})) = 2(1 - \Phi(0.4714)) = 2 \times 0.3192 = 0.638$.

Local review guidance: $\sum d^2$ shown term by term; critical value from the Spearman table with the correct tail; combined variances with squared coefficients; the doubling for an absolute difference. This is authored classroom guidance.