

Solutions

2.1 $df = (4 - 1)(3 - 1) = 6$; $E = \frac{\text{row total} \times \text{column total}}{240}$.

2.2 All six $E = 120 \times \frac{1}{6} = 20$.

2.3 Combine the three tail cells (all $E < 5$) into one ($E = 8.6$): 5 cells, $df = 5 - 1 - 1 = 3$.

3.1 (a) $E = \frac{35 \times 36}{250} = 5.04$. (b) H_0 : no association between the time of the first problem and the engine type; H_1 : there is an association. The diesel cell contributes $\frac{(10 - 5.04)^2}{5.04} = 4.881$, so $X^2 = 7.444 + 4.881 = 12.325$ on $df = (3 - 1)(3 - 1) = 4$; the 5% critical value is 9.488. Since $12.325 > 9.488$, reject H_0 – evidence of an association between when a car first experiences an engine problem and the type of engine.

3.2 (a) $\hat{p} = \frac{0(2) + 1(40) + 2(90) + 3(85) + 4(30) + 5(3)}{250 \times 5} = \frac{610}{1250} = 0.488$. (b) $r = 250 - (41.92 + 79.91 + 76.16 + 36.30 + 6.92) = 8.79$. (c) H_0 : the binomial distribution is a suitable model; H_1 : it is not. The r -class contributes $\frac{(2 - 8.79)^2}{8.79} = 5.25$, so $X^2 = 5.70 + 5.25 = 10.95$. No cells need combining here (every $E \geq 5$); the degrees of freedom are $6 - 1 - 1 = 4$, losing one for the parameter $\hat{p}$ estimated from the data. Critical value 9.488: $10.95 > 9.488$, reject – significant evidence that the binomial distribution is NOT a suitable model.

Local review guidance: expected frequencies from the correct rule (model probability or $\text{row} \times \text{col} / \text{total}$); df with the estimated parameter subtracted; rejection decided against the table value and concluded in context. This is authored classroom guidance.