

Solutions

2.1 Advantage: cheaper and faster than a census. Disadvantage: the results contain sampling error – they do not describe every member of the population.

2.2 $X_1 + 2X_2$ and $3\bar{X}$ involve sample values only: statistics. $\bar{X} - \mu$ contains the unknown population parameter μ : not a statistic.

2.3 $P(S = 0) = \frac{1}{4}$, $P(S = 1) = \frac{1}{2}$, $P(S = 2) = \frac{1}{4}$.

3.1 (a) With $p_b = \frac{2}{7}$ (large bag: constant probability), the number of black beads is $B(3, \frac{2}{7})$ and $X = 3 + 3 \cdot (\text{blacks})$: $P(X = 3) = (\frac{5}{7})^3 = \frac{125}{343}$; $P(X = 6) = \binom{3}{1} \frac{2}{7} (\frac{5}{7})^2 = \frac{150}{343}$; $P(X = 9) = \binom{3}{2} (\frac{2}{7})^2 \frac{5}{7} = \frac{60}{343}$; $P(X = 12) = (\frac{2}{7})^3 = \frac{8}{343}$ (total $\frac{343}{343}$ ✓). (b) $Y \sim B(n, \frac{60}{343})$: $P(Y \geq 1) = 1 - (\frac{283}{343})^n > 0.95$, so $(\frac{283}{343})^n < 0.05$, $n \ln \frac{283}{343} < \ln 0.05$, $n > 15.6$: least $n = 16$.

3.2 (a) $P(M = m) = P(\max = m) = \left(\frac{m}{6}\right)^2 - \left(\frac{m-1}{6}\right)^2 = \frac{2m-1}{36}$ for $m = 1..6$: $\frac{1}{36}, \frac{3}{36}, \frac{5}{36}, \frac{7}{36}, \frac{9}{36}, \frac{11}{36}$ (sums to 1 ✓). (b) $E(M) = \sum m \cdot \frac{2m-1}{36}$: summing $m(2m-1) = 2m^2 - m$ over $m = 1..6$ gives $2(91) - 21 = 161$, so $E(M) = \frac{161}{36} = 4.47$.

Local review guidance: enumeration with the correct per-draw probability and binomial weights; distribution sums to 1; follow-up counts wrapped in $B(n, p)$ with the complement taken before logs. This is authored classroom guidance.