

## 8.3 Piecewise densities and the uniform distribution

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 30 marks

**KEY WORDS** continuous uniform distribution • variance

### Piecewise densities and the uniform distribution

#### Objective

Normalise piecewise-linear densities by exact algebra (calculator-banned style), compute moments by integration, and use the uniform distribution's facts both directions (objectives 2.2, 2.4, 3.1).

#### 1 Key knowledge and worked examples

**Two shapes.** A piecewise-linear pdf normalises to (usually) a perfect square in the join point  $k$  – factorise it. Uniform on  $[a, b]$ : density  $\frac{1}{b-a}$ , mean  $\frac{a+b}{2}$ , variance  $\frac{(b-a)^2}{12}$ ,  $E(X^2) = \text{Var} + \text{mean}^2$ ; probabilities are lengths over  $b-a$ ; an interval probability gives  $b-a$ , the mean gives  $a+b$ .

**Worked example – the two shapes (WST02 June 2025 Q5, Q6).** (i)  $f = \frac{2x}{21}$  on  $[0, k]$  then  $\frac{2(6-x)}{15}$  on  $(k, 6]$ : normalising gives  $4k^2 - 28k + 49 = 0$ , i.e.  $(2k-7)^2 = 0$ , so  $k = 3.5$ ; then  $E(X) = \frac{19}{6}$  and (with  $E(X^2) = \frac{277}{24}$ )  $\text{Var} = \frac{109}{72}$ . (ii) Uniform with  $P(2 < X < 5) = \frac{3}{16}$  and mean 6:  $b-a = 16$ ,  $a+b = 12 \rightarrow a = -2$ ,  $b = 14$ ; variance  $\frac{64}{3}$ ,  $E(X^2) = \frac{172}{3}$ ,  $P(\frac{3}{2}X - b > a) = P(X > 8) = \frac{3}{8}$ .

#### 2 Practice

**2.1**  $X$  is uniform on  $[3, 11]$ . Write down the mean, the variance and  $P(5 < X < 9)$ . [3]

**2.2**  $X$  is uniform on  $[a, b]$  with mean 10 and variance  $\frac{64}{3}$ . Find  $a$  and  $b$ . [3]

**2.3**  $f(x) = \frac{x}{c}$  on  $[0, 4]$ . Find  $c$  and  $E(X)$ . [3]

### 3 Exam-style questions

**3.1** (*Edexcel IAL WST02 June 2025 Q5.*) The continuous random variable  $X$  has probability density function  $f(x) = \frac{2x}{21}$  for  $0 \leq x \leq k$  and  $f(x) = \frac{2(6-x)}{15}$  for  $k < x \leq 6$  (0 otherwise), where  $k$  is a positive constant. Given that  $E(X^2) = \frac{277}{24}$ : (a) show, with all working, that  $k = 3.5$ ; (b) use algebraic integration to find  $E(X)$ ; (c) find  $\text{Var}(X)$ . [11]

**3.2** (*Edexcel IAL WST02 June 2025 Q6.*) The continuous random variable  $X$  is uniformly distributed on the interval  $[a, b]$ . Given that  $P(2 < X < 5) = \frac{3}{16}$  and  $E(X) = 6$ : (a) find the value of  $a$  and the value of  $b$ ; (b) find the value of the constant  $c$  for which  $E(cX + 1) = 3$ ; (c) write down the exact value of  $\text{Var}(X)$ ; (d) find the exact value of  $E(X^2)$ ; (e) find  $P(\frac{3}{2}X - b > a)$ . [10]

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