

Solutions

2.1 $P(Z > 1.5) = 0.0668$; by symmetry $P(Z < -1.5) = 0.0668$.

2.2 $z = \frac{46-40}{6} = 1$: $P(X > 46) = P(Z > 1) = 0.1587$.

2.3 $P\left(Z > \frac{56-50}{\sigma}\right) = 0.1587 \implies z = 1 \implies \sigma = 6$.

3.1 (a) $P(X > 508) = P\left(Z > \frac{508-502}{3}\right) = P(Z > 2) = 0.0228$. Since 496 and 508 sit symmetrically two SDs either side of 502, $P(496 < X < 508) = 1 - 2(0.0228) = 0.9544$. (b) With $p = 0.0228$: $\binom{4}{2}p^2(1-p)^2 = 6(0.0228)^2(0.9772)^2 = 0.00298$. (c) The 85th percentile has $z = 1.0364$: $\frac{1038.51-1024}{\sigma} = 1.0364$ gives $\sigma = \frac{14.51}{1.0364} = 14.0$, i.e. $\sigma = 14$ ✓.

3.2 Standardise both: $P(X > k) = P\left(Z > \frac{k-502}{3}\right)$ and $P(Y < 2k) = P\left(Z < \frac{1024-2k}{14}\right)$. For these two probabilities to be equal, the two z -values must be EQUAL – a right tail and a left tail carry the same probability exactly when they are the same distance from 0 in the same direction: $\frac{k-502}{3} = \frac{1024-2k}{14}$. Clearing: $14k - 7028 = 3072 - 6k$, so $20k = 10100$ and $k = 505$. Then $p = P\left(Z > \frac{505-502}{3}\right) = P(Z > 1) = 0.159$.

Local review guidance: standardisations with the correct sign pairings (equal probabilities $\implies$ equal z for a $>$ and a $<$ on opposite sides); exact table values with 4 dp tolerance; binomial term with the correct coefficient. This is authored classroom guidance.