

# 7.5 Discrete random variables

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ Total: 23 marks

KEY WORDS discrete random variable • expectation

## Discrete random variables

### Objective

Use probability functions and cumulative functions of discrete random variables, compute  $E(X)$ ,  $E(X^2)$ ,  $\text{Var}(X)$ , apply linear coding, and quote the discrete uniform results (objectives 5.1–5.4).

### 1 Key knowledge and worked examples

**Routine.** Probabilities sum to 1 (find missing values).  $F(x_0) = P(X \leq x_0)$ . Solve inequalities for X BEFORE summing probabilities.  $E(X) = \sum xp(x)$ ,  $E(X^2) = \sum x^2p(x)$ ,  $\text{Var}(X) = E(X^2) - [E(X)]^2$ ;  $E(aX + b) = aE(X) + b$ ,  $\text{Var}(aX + b) = a^2\text{Var}(X)$ . Uniform on 1..n:  $E(X) = \frac{n+1}{2}$ ,  $\text{Var}(X) = \frac{n^2-1}{12}$ .

**Worked example – the full routine (WST01 June 2025 Q1).** X on 1–4 with probabilities 0.3, 0.2, 0.35, 0.15:  $F(3.5) = 0.85$ ;  $3X - 3 < X + 2 \Rightarrow X < 2.5 \Rightarrow P = 0.5$ ;  $E(X) = 2.35$ ;  $E(X^2) = 6.65$ ;  $\text{Var}(X) = 1.1275$ ;  $\text{Var}(5 - 2X) = 4(1.1275) = 4.51$ .

### 2 Practice

2.1 X has  $P(X = x) = \frac{kx}{6}$  for  $x = 1, 2, 3$ . Find  $k$ ,  $E(X)$  and  $\text{Var}(X)$ . [4]

2.2 A fair die is rolled once. Using the uniform results, write down  $E(X)$  and  $\text{Var}(X)$ . [2]

2.3 Y has  $E(Y) = 4$  and  $\text{Var}(Y) = 2.5$ . Find  $E(3Y - 2)$  and  $\text{Var}(3Y - 2)$ . [2]

### 3 Exam-style questions

3.1 (Edexcel IAL WST01 June 2025 Q1.) The discrete random variable X has probability distribution

| $x$        | 1   | 2   | 3    | 4    |
|------------|-----|-----|------|------|
| $P(X = x)$ | 0.3 | 0.2 | 0.35 | 0.15 |

(a) Find  $F(3.5)$ . (b) Find  $P(3X - 3 < X + 2)$ . (c) Find  $E(X)$ . (d) Show that  $\text{Var}(X) = 1.1275$ . (e) Find  $\text{Var}(5 - 2X)$ . [9]

3.2 (Original.) A game scores X where  $P(X = 0) = 0.25$ ,  $P(X = 1) = 0.5$  and  $P(X = 4) = 0.25$ . The winnings are  $W = 10X - 3$ . (a) Find  $E(W)$  and  $\text{Var}(W)$ . (b) Find  $P(W > E(W))$ . [6]