

# 6.5 Vertical circular motion

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 23 marks**

**KEY WORDS** vertical circle

## Vertical circular motion

### Objective

Combine energy with Newton’s law towards the centre for vertical circles: speeds at levels, leaving conditions ( $R = 0$ ,  $T = 0$ ) and projectile continuation conserving the horizontal component (objective 4.4).

### 1 Key knowledge and worked examples

**The narrative.** Energy between levels gives  $v^2$  at each point; Newton’s law towards the centre relates  $v^2$  to the forces; the leaving condition on a surface is  $R = 0$  (on a string,  $T = 0$ ). After leaving, the particle is a projectile whose horizontal velocity component is conserved.

**Worked example (WME03 June 2025 Q5).** P released at A on a smooth sphere (radius  $r$ ), OA at  $\alpha$  to the upward vertical; leaves at B with OB at  $\alpha$  to the horizontal. (a) A is  $r \cos \alpha$  above O and B is  $r \sin \alpha$  above O, so  $\frac{1}{2}mv^2 = mgr(\cos \alpha - \sin \alpha)$ . (b) At B,  $R = 0$ :  $mg \sin \alpha = \frac{mv^2}{r} \implies \sin \alpha = 2(\cos \alpha - \sin \alpha) \implies \tan \alpha = \frac{2}{3}$ . (c) After B, conserve  $v \sin \alpha$  horizontally; at the horizontal through O,  $\frac{1}{2}mW^2 = mgr \cos \alpha$ , and  $\cos^2 \theta = \frac{v^2 \sin^2 \alpha}{W^2} = \frac{4}{39}$ .

### 2 Practice

**2.1** A particle on a string (length  $r$ ) swings in a vertical circle. Write the condition for completing the circle when the speed at the top is just sufficient. [2]

**2.2** A bead slides from rest at the top of a smooth sphere of radius  $a$ . Show it leaves the surface when  $\cos \theta = \frac{2}{3}$  ( $\theta$  from the upward vertical). [4]

**2.3** A particle is projected at speed  $\sqrt{4ga}$  from the lowest point of a smooth cylinder (radius  $a$ , outside surface). Find where it leaves. [3]

### 3 Exam-style questions

**3.1** (*Edexcel IAL WME03 June 2025 Q5.*) A fixed solid sphere has centre O and radius  $r$ . A particle P of mass  $m$  is held at rest at A on the smooth outer surface, where OA makes an angle  $\alpha$  ( $\alpha < 45^\circ$ ) with the upward vertical. P is released from rest and leaves the surface at B, where OB makes an angle  $\alpha$  with the horizontal, moving with speed  $v$ . Air resistance is negligible. (a) Show that  $v^2 = 2gr(\cos \alpha - \sin \alpha)$ . (b) Show that  $\tan \alpha = \frac{2}{3}$ . (c) At the instant P crosses the horizontal plane through O it moves at angle  $\theta$  to the horizontal. Show that  $\cos \theta = \frac{2}{\sqrt{39}}$ . [14]