

Solutions

2.1 At the top, gravity alone provides the centripetal force: $mg = \frac{mv_{top}^2}{r}$, i.e. $v_{top}^2 = gr$; the speed at the bottom must then be $\sqrt{5gr}$ by energy.

2.2 Energy from the top: $\frac{1}{2}mv^2 = mga(1 - \cos \theta)$. Leaving ($R = 0$): $mg \cos \theta = \frac{mv^2}{a}$, so $\cos \theta = 2(1 - \cos \theta)$ and $3 \cos \theta = 2$ ✓.

2.3 Measure θ from the upward vertical. The particle climbs from the bottom ($\theta = 180^\circ$) to height $a(1 + \cos \theta)$, so energy from launch gives $v^2 = 4ga - 2ga(1 + \cos \theta) = 2ga(1 - \cos \theta)$. On the outside of the cylinder the inward direction points towards the axis; the weight's component along it is $mg \cos \theta$ and the reaction R points outward: $mg \cos \theta - R = \frac{mv^2}{a}$. The particle leaves when $R = 0$: $ga \cos \theta = 2ga(1 - \cos \theta)$, so $\cos \theta = \frac{2}{3}$ – it leaves at $\theta \approx 48^\circ$ from the top, having climbed $\frac{5}{3}a$, still with $v^2 = \frac{2ga}{3} > 0$ ✓.

3.1 (a) The height of A above O is $r \cos \alpha$; the height of B above O is $r \sin \alpha$; P falls $r(\cos \alpha - \sin \alpha)$, so $\frac{1}{2}mv^2 = mgr(\cos \alpha - \sin \alpha)$ ✓. (b) At B the reaction is zero and the component of weight towards O is $mg \sin \alpha$ (OB at α to the horizontal): $mg \sin \alpha = \frac{mv^2}{r}$. Substituting (a): $\sin \alpha = 2(\cos \alpha - \sin \alpha)$, so $3 \sin \alpha = 2 \cos \alpha$ and $\tan \alpha = \frac{2}{3}$ ✓. (c) From (b), $\cos \alpha = \frac{3}{\sqrt{13}}$, $\sin \alpha = \frac{2}{\sqrt{13}}$, and $v^2 = \frac{2gr}{\sqrt{13}}$. After B the horizontal component $v \sin \alpha$ is conserved: $v^2 \sin^2 \alpha = \frac{8gr}{13\sqrt{13}}$. At the horizontal through O, energy from A: $\frac{1}{2}mW^2 = mgr \cos \alpha$, so $W^2 = \frac{6gr}{\sqrt{13}}$. Then $\cos^2 \theta = \frac{v^2 \sin^2 \alpha}{W^2} = \frac{8}{78} = \frac{4}{39}$, giving $\cos \theta = \frac{2}{\sqrt{39}}$ ✓.

Local review guidance: heights measured from the centre with the stated angles; the $R = 0$ substitution; (dM1) horizontal-component conservation after leaving. This is authored classroom guidance.