

6.3 Simple harmonic motion

Name: _____ Class: _____ Date: _____ Total: 25 marks

KEY WORDS simple harmonic motion • period

Simple harmonic motion

Objective

Prove SHM by deriving $\ddot{x} = -\omega^2 x$ (equilibrium tensions first), quote period and amplitude results, and compute exact time fractions (objectives 3.2, 3.3).

1 Key knowledge and worked examples

SHM routine. Find the equilibrium point; at displacement x write the net force with Hooke’s law for every string; collect terms in x : $\ddot{x} = -\omega^2 x$ proves SHM. Period $\frac{2\pi}{\omega}$; amplitude a ; $v^2 = \omega^2(a^2 - x^2)$; $v_{max} = a\omega$; $x = a \cos \omega t$ (released from an end). Time fractions come from $|\sin \omega t|$ or $|\cos \omega t|$ inequalities.

Worked example (WME03 June 2025 Q7). Strings from P to A ($\lambda = mg$, natural $3L$) and B ($\lambda = 2mg$, natural $2L$), $AB = 7L$: equilibrium at E ($AE = 4.5L$, both tensions $\frac{mg}{2}$). At displacement x from E: $m\ddot{x} = \frac{2mg}{2L}(\frac{L}{2} - x) - \frac{mg}{3L}(\frac{3L}{2} + x) = -\frac{4mg}{3L}x$: SHM with $\omega^2 = \frac{4g}{3L}$, period $\pi\sqrt{\frac{3L}{g}}$; amplitude $\frac{L}{2}$ (from AC = 5L), so $v_{max} = \sqrt{\frac{gL}{3}}$; the time with $|v| \leq \sqrt{\frac{gL}{12}} = \frac{v_{max}}{2}$ is one third of a period, $\pi\sqrt{\frac{L}{3g}}$.

2 Practice

2.1 A particle on a smooth table is attached to a spring (λ , natural L) fixed at A, and rests at equilibrium at E. It is displaced a small distance and released: write down ω^2 and the period. [3]

2.2 For SHM with amplitude 0.4 m and $\omega = 3 \text{ rad s}^{-1}$, find the maximum speed and the speed when $x = 0.2 \text{ m}$. [3]

2.3 For that oscillation, find the time per oscillation for which $|x| \leq 0.2 \text{ m}$. [3]

3 Exam-style questions

3.1 (*Edercel IAL WME03 June 2025 Q7.*) A particle P of mass m lies at rest on a smooth horizontal table. One end of a light elastic string of natural length $3L$ and modulus of elasticity mg is attached to P and the other to a fixed point A on the table; a second string of natural length $2L$ and modulus $2mg$ runs from P to a fixed point B, with A, P, B collinear and $AB = 7L$. P rests in equilibrium at E with $AE = 4.5L$. P is held at C on AB with $AC = 5L$ and released from rest. (a) Show that P moves with simple harmonic motion about E and find the period. (b) Find the maximum speed of P. (c) Find the exact time, in any one oscillation, for which the speed of P is less than or equal to $\sqrt{gL/12}$. [16]

[16]