

Solutions

2.1 Moments about the left support: $R_R(6) = 200(3) + 500(2) = 1600$, so $R_R = 266.7$ N; $R_L = 700 - 266.7 = 433.3$ N.

2.2 The ground reaction $R = \text{total vertical force} = 120$ N (the smooth wall gives only a horizontal force). So $\mu = \frac{48}{120} = 0.4$.

2.3 Moments about the foot: $S_{\text{wall}}(5 \sin 60^\circ) = 10g(2.5 \cos 60^\circ)$, so $S_{\text{wall}} = \frac{10(9.8)(1.25)}{5(0.866)} = 28.3$ N.

3.1 (a) The rail is smooth, so its reaction S is perpendicular to the rod at C. Moments about A (eliminates the floor forces): the weight mg acts at the midpoint, distance a along the rod, with moment arm $a \cos \theta$; S acts at distance $1.5a$ with full moment arm: $S(1.5a) = mg a \cos \theta$, so $S = \frac{2mg \cos \theta}{3}$ ✓. (b) Vertical equilibrium: S is perpendicular to the rod, so at rod angle θ its vertical component is $S \cos \theta$ and $V = mg - \frac{2mg \cos \theta}{3} \cos \theta = mg - \frac{2mg}{3} \cos^2 \theta = \frac{mg}{3}(3 - 2 \cos^2 \theta)$ ✓. (c) Horizontal equilibrium: $F = S \sin \theta$. Limiting: $F = \frac{4}{7}V$. So $\frac{4}{7} \cdot \frac{mg}{3}(3 - 2 \cos^2 \theta) = \frac{2mg}{3} \cos \theta \sin \theta$. Divide by $\frac{mg}{3} \cos^2 \theta$: $\frac{4}{7}(3 \sec^2 \theta - 2) = 2 \tan \theta$. With $\sec^2 \theta = 1 + \tan^2 \theta$: $\frac{4}{7}(1 + 3 \tan^2 \theta) = 2 \tan \theta$, so $4 + 12 \tan^2 \theta = 14 \tan \theta$, i.e. $6 \tan^2 \theta - 7 \tan \theta + 2 = 0$ ✓.

3.2 Moments about the foot: $S(6 \sin 50^\circ) = 12g(3 \cos 50^\circ)$, so $S = \frac{12(9.8)(3)(0.6428)}{6(0.766)} = \frac{226.7}{4.596} = 49.3$ N. Horizontal: $F = S = 49.3$; vertical: $R = 12g = 117.6$ N. So $\mu = \frac{49.3}{117.6} = 0.419$ (0.419).

Local review guidance: moments about the pivot that eliminates the contact unknowns with correct moment arms (components of distances, not the distances); components of the perpendicular reaction resolved with the right angle; (DM1) the $\sec^2 \theta$ substitution completing the $\tan \theta$ equation. This is authored classroom guidance.