

# 5.2 Centres of mass

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 23 marks**

**KEY WORDS** centre of mass • wire framework

## Centres of mass

### Objective

Find centres of mass of particle systems, uniform laminas, composite shapes (negative areas) and wire frameworks (length weighting with signed distances), and use suspension geometry (objectives 2.1–2.3).

### 1 Key knowledge and worked examples

**Table method.** Weight each piece by mass – for a uniform lamina by AREA, for a wire by LENGTH – and take moments about a convenient line:  $\sum(\text{weight})(\bar{x}_{\text{piece}}) = (\text{total})(\bar{x})$ . Holes and opposite-side pieces enter NEGATIVE. Booklet results: semi-circular wire  $\frac{2r}{\pi}$ , semicircular lamina  $\frac{4r}{3\pi}$  from the centre. Suspended from a point, the body hangs with G vertically below it; the asked angle usually splits a triangle angle at the suspension point.

**Worked example – an S-shaped framework (WME02 June 2025 Q3).** Arcs AB ( $r = 2a$ , length  $2\pi a$ , COM  $\frac{4a}{\pi}$  one side) and CD ( $r = a$ , length  $\pi a$ , COM  $\frac{2a}{\pi}$  the OTHER side) joined by straights  $AC = DB = a$  on the line: moments about the line give  $2\pi a \cdot \frac{4a}{\pi} - \pi a \cdot \frac{2a}{\pi} = 6a^2$ ; total length  $(3\pi + 2)a$ ; so  $\bar{x} = \frac{6a}{3\pi+2}$ . Suspended from A with E on arc AB at  $\angle AOE = 90^\circ$ : G lies on OE,  $\tan \angle OAG = \frac{3}{3\pi+2}$ , and  $\theta = 45^\circ - \angle OAG$  (since  $OA = OE$ ), giving  $\tan \theta = \frac{3\pi-1}{3\pi+5}$ .

### 2 Practice

**2.1** Particles of mass 2 kg, 3 kg and 5 kg sit at  $x = 0, 4$  and 6 on the x-axis. Find their centre of mass. [2]

**2.2** A uniform rectangular lamina 6 m by 4 m has a 2 m by 2 m square hole cut from one corner. Find the distance of the centre of mass from each outer edge adjacent to that corner. [4]

**2.3** Three uniform rods form a uniform wire triangle with sides 5, 5 and 6. Find the distance of the centre of mass from the 6-side. [3]

**3.1** (*Edexcel IAL WME02 June 2025 Q3, condensed.*) A uniform rigid wire framework consists of two semicircular arcs AB and CD, both of centre O, joined by two straight pieces AC and DB. The arc AB has radius  $2a$ , the arc CD has radius  $a$ ,  $AC = DB = a$ , and ACDB is a straight line. The arcs lie on opposite sides of ACDB. The centre of mass of a semicircular arc of radius  $r$  is  $\frac{2r}{\pi}$  from its centre. (a) Show that the centre of mass of the framework is  $\frac{6a}{3\pi+2}$  from AB. (b) The point E lies on the arc AB with  $\angle AOE = 90^\circ$ . The framework is freely suspended from A and hangs in equilibrium with AE at angle  $\theta$  to the downward vertical. Show that  $\tan \theta = \frac{3\pi - 1}{3\pi + 5}$ . [10]

**3.2 (Original.)** A uniform semicircular lamina of radius 6 cm is freely suspended from one end P of its straight edge. Find the angle between the straight edge and the vertical, giving an exact tangent. [4]