

Solutions

2.1 $\bar{x} = \frac{2(0)+3(4)+5(6)}{10} = \frac{42}{10} = 4.2$ from the 2 kg particle.

2.2 Rectangle (area 24) at (3, 2); hole (area -4) at (1, 1) from the adjacent corner:
 $\bar{x} = \frac{24(3)-4(1)}{20} = \frac{68}{20} = 3.4$ m and $\bar{y} = \frac{24(2)-4(1)}{20} = \frac{44}{20} = 2.2$ m from the two outer edges at that corner.

2.3 The two 5-sides each have midpoint 1.5 m above the 6-side (height of the 5-5-6 triangle = 4); the 6-side has COM distance 0. Weights by length: $\bar{y} = \frac{5(1.5)+5(1.5)+6(0)}{16} = \frac{15}{16} = 0.9375$ m from the 6-side.

3.1 (a) Table about the line AB: arc AB length $2\pi a$, COM $\frac{4a}{\pi}$ above; arc CD length πa , COM $\frac{2a}{\pi}$ BELOW; straights total $2a$ on the line: $2\pi a \cdot \frac{4a}{\pi} - \pi a \cdot \frac{2a}{\pi} + 2a \cdot 0 = 8a^2 - 2a^2 = 6a^2$; total length $= (3\pi + 2)a$; so $\bar{x} = \frac{6a^2}{(3\pi + 2)a} = \frac{6a}{3\pi + 2}$ ✓ (the COM is on the AB side, distance d from the line). (b) By symmetry G lies on OE. In triangle AOE, $OA = OE = 2a$ and the angle at O is 90° , so $\angle OAE = 45^\circ$; the COM is at distance d from A's vertical: in right triangle AOG, $\tan \angle OAG = \frac{d}{2a} = \frac{6}{2(3\pi + 2)} = \frac{3}{3\pi + 2}$. Hanging from A puts G on the vertical, so $\theta = \angle GAE = \angle OAE - \angle OAG = 45^\circ - \angle OAG$, and $\tan \theta = \frac{1 - \frac{3}{3\pi + 2}}{1 + \frac{3}{3\pi + 2}} = \frac{3\pi - 1}{3\pi + 5}$ ✓.

3.2 The COM sits on the symmetry axis at $\frac{4r}{3\pi} = \frac{8}{\pi}$ cm from the centre O, i.e. $\frac{8}{\pi}$ from the straight edge, on the perpendicular through O. Suspended from P (one end of the edge, $OP = 6$), G hangs below P. In the right triangle with legs $OP = 6$ (along the edge) and the COM's perpendicular distance $\frac{8}{\pi}$ from the edge: the angle θ between the edge and the vertical satisfies $\tan \theta = \frac{8/\pi}{6} = \frac{4}{3\pi}$.

Local review guidance: pieces weighted by area/length with the correct booklet distance; opposite-side pieces signed; suspension angles via right-triangle trigonometry at the suspension point plus isosceles geometry. This is authored classroom guidance.