Solutions

2.1 Circle, centre $2i$ (the point $(0, 2)$), radius 3.

2.2 Perpendicular bisector of the points 3 and -1: the vertical line $\operatorname{Re} z = 1$.

2.3 With $z = x+iy$: $(x-2)^2+y^2 = 4\left[(x+1)^2+y^2\right]$, so $x^2-4x+4+y^2 = 4x^2+8x+4+4y^2$, i.e. $3x^2+12x+3y^2 = 0$, so $x^2+4x+y^2 = 0$, i.e. $(x+2)^2+y^2 = 4$: circle, centre $(-2, 0)$, radius 2. (Check: $z = 0$ gives $|-2| = 2|1|$ ✓ and 0 lies on it.)

2.4 $\arg\left(\frac{z-1}{z+1}\right) = \arg(z-1) - \arg(z+1)$ is the angle at z subtended by the segment from -1 to 1. Equal to $\frac{\pi}{2}$: the semicircle $x^2+y^2 = 1$, $y > 0$, excluding the endpoints ± 1.

3.1 $w = \frac{1}{z} \Rightarrow z = \frac{1}{w}$. Then $|\frac{1}{w} - 2| = 2$ gives $|1-2w| = 2|w|$. Squaring: $1-2w-2\bar{w}+4|w|^2 = 4|w|^2$, so $w+\bar{w} = \frac{1}{2}$, i.e. $\operatorname{Re} w = \frac{1}{4}$ – a vertical line in the w-plane, $\frac{1}{4}$ unit to the right of the imaginary axis. (Check: $z = 4$ is on the circle, and $z = 4 \mapsto w = \frac{1}{4}$ ✓.)

3.2 (a) Unit circle, centre the origin. (b) $z = \dfrac{9i-w}{w+i}$, so $|9i-w| = |w+i| \cdot |z| = |w+i|$. With $w = u+iv$: $9i-w = -u+i(9-v)$, so $u^2+(9-v)^2 = u^2+(v+1)^2$, giving $81-18v = 2v+1$, i.e. $20v = 80$: l is $v = 4$. (c) $z = i$: $w = \dfrac{9i-i^2}{1+i} = \dfrac{1+9i}{1+i} = \dfrac{(1+9i)(1-i)}{2} = \dfrac{10+8i}{2} = 5+4i$, whose imaginary part is 4, so it lies on l ✓.

Local review guidance: z made the subject before any modulus work; modulus equation expanded in u, v; line/circle description with geometry named (bisector, centre, radius). This is authored classroom guidance.