

Solutions

2.1 Pivot (position 2 of 3) = 9: less = 5, 2, 6; greater = none $\rightarrow$ 5, 2, 6, **9**. Sublist 5, 2, 6 (pivot 2): less none; greater = 5, 6 $\rightarrow$ **2**, 5, 6. Sublist 5, 6 (pivot 5): less none; greater = 6 $\rightarrow$ **5**, 6. Final: 2, 5, 6, 9.

2.2 From R: edges RS 4, RT 3 – take RT(3). From {R,T}: edges to new vertices are RS 4, TS 1, TU 2 – take TS(1). From {R,T,S}: edges to U are SU 5 and TU 2 – take TU(2). MST = RT + TS + TU, total 6.

2.3 Vertices: (0,0): 0; (5,0) ($2x + y = 10$): 5; (2,6) (intersection: $2x + y = 10, x + y = 8 \rightarrow x = 2, y = 6$): $2 + 12 = 14$; (0,8): 16. Maximum 16 at (0,8).

3.1 (a) Total = $4 + 2 + 3 + 5 + 6 + 2 + 4 + 1 = 27$. (b) Degrees: P 3 (PQ, PR, PS), Q 3 (PQ, QR, QT), R 4 (QR, RS, PR, RT), S 3 (RS, ST, PS), T 3 (ST, QT, RT): odd villages P, Q, S, T. (c) Pairings of {P, Q, S, T} (weights are shortest paths where no direct edge exists): $PQ + ST = 4 + 5 = 9$; $PS + QT = 2 + 4 = 6$; $PT + QS = 7 + 5 = 12$ (PT via PS–ST or PR–RT, both 7; QS via QR–RS, 5). Least = $PS + QT = 6$. (d) Route length = $27 + 6 = 33$.

3.2 (a) Forward pass: A 0–2, F 0–3 (F after A: starts 2, ends 3), B 0–5, C starts 2 ends 5, D starts 5 ends 9, E starts 9 ends 11. Duration 11; critical path B–D–E ($5 + 4 + 2 = 11$). (b) F's latest finish = latest start of nothing downstream (F is an end activity): latest finish 11, earliest start 2, duration 1: float = $11 - 2 - 1 = 8$ days. (c) Vertices of $\{x \leq 3, y \leq 2, x + y \leq 4\}$: (0,0), (3,0), (2,2), (0,2): $W = 0, 3, 4, 2$: optimum $W = 4$ at (2,2) – already integral ✓.

Local review guidance: pairing weights use shortest paths when the direct edge is absent; end activities' latest finish is the project duration; integer optima confirmed at neighbouring lattice points. This is authored classroom guidance.