

Solutions

2.1 Sorted: AC 1, BC 2, AB 3, CD 4, AD 5. Take AC(1) ✓; BC(2) ✓ (B joins); AB(3) – cycle (A–C–B) reject; CD(4) ✓ (D joins); AD(5) – cycle reject. MST = AC + BC + CD, total 7.

2.2 Prim from B: cheapest edge from {B} is BC(2); from {B,C} cheapest to a new vertex is CA(1); from {B,C,A} cheapest to D is CD(4). Same tree, total 7 ✓.

2.3 A = 0 permanent. Neighbours: B 2, C 2. B permanent (2) – D has no edge from B, so no update. C permanent (2; the tie with B may be broken in either order); update D: $2 + 1 = 3$, smaller than the direct AD 4. D permanent (3): shortest A to D = 3, route A–C–D ($2 + 1 = 3 < 4$ ✓).

3.1 (a) Sorted: QR 2, PR 3, QS 5, RS 6, PQ 7, PS 8. Take QR(2) ✓; PR(3) ✓ (P joins); QS(5) ✓ (S joins); RS(6) – cycle reject; PQ(7) – cycle reject; PS(8) – cycle reject. MST = QR + PR + QS, total 10. (b) Prim from P: PR(3); from {P,R} cheapest new edge RQ(2); from {P,R,Q} cheapest to S is QS(5). Same tree, total 10 ✓. (c) Both construct a spanning tree of least total weight; any MST of a network has the same minimum total, so the totals agree (the edges can differ only when weights tie).

3.2 P = 0 permanent. Update: R 3, Q 7, S 8. R permanent (3); update Q: $3 + 2 = 5$ (smaller than 7), S: $3 + 6 = 9$ (8 stays). Q permanent (5); update S via Q: $5 + 5 = 10 > 8$, so S keeps 8. S permanent (8). Shortest P to S = 8, route P–S (the alternatives P–R–S = $3 + 6 = 9$ and P–R–Q–S = $3 + 2 + 5 = 10$ are both longer ✓).

Local review guidance: edges chosen in weight order with cycle rejections shown; Prim grows by cheapest new-vertex edge, not cheapest free edge; Dijkstra labels in permanence order with updates shown. This is authored classroom guidance.