# 1.5 Parabola and rectangular hyperbola

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

**KEY WORDS** parabola • rectangular hyperbola • focus • directrix • normal

## Parabola and rectangular hyperbola

## Objective

Use the Cartesian and parametric equations of the parabola $y^2 = 4ax$ and rectangular hyperbola $xy = c^2$, their focus–directrix property, and tangents/normals by Cartesian differentiation (FP1 4.1–4.4).

## 1 Key knowledge and worked examples

**Standard curves.** Parabola: $y^2 = 4ax$, parametric $(at^2, 2at)$; focus $(a, 0)$, directrix $x = -a$. Rectangular hyperbola: $xy = c^2$, parametric $(ct, \frac{c}{t})$. Gradients by Cartesian differentiation: $y^2 = 4ax \Rightarrow \frac{dy}{dx} = \frac{2a}{y} = \frac{1}{t}$ at parameter $t$; $xy = c^2 \Rightarrow \frac{dy}{dx} = -\frac{c^2}{x^2} = -\frac{1}{t^2}$.

**Worked example – normal to a hyperbola (WFM01 June 2025 Q4).** For $xy = 81$ at $P(9t, \frac{9}{t})$: $\frac{dy}{dx} = -\frac{81}{x^2} = -\frac{1}{t^2}$, so the normal has gradient $t^2$ and equation $y - \frac{9}{t} = t^2(x - 9t)$, i.e. $ty = t^3x + 9(1 - t^4)$. At the $y$-axis, $A = (0, \frac{9(1-t^4)}{t})$. For $t = \frac{1}{3}$: $P = (3, 27)$, $A = (0, \frac{80}{3})$ and triangle $OPA$ has area $\frac{1}{2} \times 3 \times \frac{80}{3} = 40$.

**Worked example – focus-directrix distances (WFM01 June 2025 Q9).** $P$ on a parabola is 5 units from the focus, hence 5 units from the directrix ($PF = PM$). Points $Q, R$ on the directrix have $PQ = PR = 10$. Right triangle $PQM$: $QM = \sqrt{100 - 25} = 5\sqrt{3}$, and $M$ lies between $Q$ and $R$, so $QR = 10\sqrt{3}$.

**Worked example – tangent to a parabola (original).** $y^2 = 12x$ at parameter $t = 2$ gives the point $(at^2, 2at) = (12, 12)$ and gradient $\frac{1}{t} = \frac{1}{2}$: tangent $y - 12 = \frac{1}{2}(x - 12)$, i.e. $x - 2y + 12 = 0$. Check: $144 = 12 \times 12$ ✓ on the curve.

## 2 Practice

**2.1** The parabola $C$ has equation $y^2 = 8x$. Write down the coordinates of its focus and the equation of its directrix, and the parametric coordinates of a general point. [3]

**2.2** For $y^2 = 20x$ at the point with $t = 3$, find the gradient of the tangent and its equation. [3]

**2.3** The rectangular hyperbola $xy = 16$ has a point $Q$ with $x = 8$. Find $\frac{dy}{dx}$ at $Q$ and the equation of the normal at $Q$. [4]

**2.4** A point $P$ on the parabola $y^2 = 4ax$ is 7 units from its focus. Find the distance from $P$ to the directrix and the $x$-coordinate of $P$. [3]

## 3 Exam-style questions

**3.1** *(Adapted from Edexcel IAL WFM01 June 2025 Q4.)* The rectangular hyperbola $H$ has equation $xy = 36$; the point $P(6t, \frac{6}{t})$, $t \neq 0$, lies on $H$. (a) Use calculus to show that the normal to $H$ at $P$ has equation $ty = t^3x + 6(1 - t^4)$. (b) The normal meets the $y$-axis at $A$; find the exact coordinates of $A$ in terms of $t$. (c) Given $t = \frac{1}{2}$, find the exact area of triangle $OPA$. [8]

**3.2** *(Adapted from Edexcel IAL WFM01 June 2025 Q9.)* Figure: a parabola with focus $F$, directrix $d$, and a point $P$ on the curve. The points $Q$ and $R$ lie on $d$ with $PQ = PR = 12$, and $P$ is 6 units from $F$. Determine the exact length $QR$. [4]