Solutions

2.1 (a) $8+5i$; (b) $8-6i+4i-3i^2 = 11-2i$; (c) $\dfrac{(5+i)(3+i)}{10} = \dfrac{15+5i+3i+i^2}{10} = \dfrac{14+8i}{10} = \frac{7}{5}+\frac{4}{5}i$.

2.2 (a) $|z| = 5$, $\arg z = \arctan\frac{4}{3} \approx 0.927$; (b) $2\sqrt{2}$, $\frac{3\pi}{4}$; (c) 2, $\tan\theta = \sqrt{3}$ with both parts negative so $\theta = -\frac{2\pi}{3}$ (or $\frac{4\pi}{3}$).

2.3 $(x+2)^2 + 25 = 0 \Rightarrow x = -2 \pm 5i$.

2.4 $|z_1 z_2| = \sqrt{2}\cdot 2 = 2\sqrt{2}$. In Cartesian form $z_1 = 1+i$ and $z_2 = -\sqrt{3}+i$, so $z_1 z_2 = -(1+\sqrt{3}) + (1-\sqrt{3})i$ and $|z_1 z_2|^2 = (1+\sqrt{3})^2 + (1-\sqrt{3})^2 = 8$, confirming $2\sqrt{2}$.

2.5 z at $(2,3)$; $\bar{z}$ at $(2,-3)$: reflection in the real axis; $iz = -3+2i$: quarter-turn anticlockwise about O; $-z = -2-3i$: half-turn about O.

3.1 (a) $w = 4(-\frac{1}{2}+\frac{\sqrt{3}}{2}i) = -2+2\sqrt{3}\,i$. (b) $|w^2| = 16$. (c) w in the second quadrant on the circle of radius 4; w^2 on the circle of radius 16 – multiplying scales the modulus (and turns the point through $2\times\frac{2\pi}{3} = \frac{4\pi}{3}$, equivalent to $-\frac{2\pi}{3}$, landing in the third quadrant).

3.2 $z_1 = 3(-\frac{\sqrt{3}}{2}+\frac{1}{2}i) = -\frac{3\sqrt{3}}{2}+\frac{3}{2}i$. Imaginary parts cancel: $b = -\frac{3}{2}$. Then $a^2+\frac{9}{4} = |z_2|^2 = (6/3)^2 = 4$, so $a = \pm\frac{\sqrt{7}}{2}$: $z_2 = \pm\frac{\sqrt{7}}{2} - \frac{3}{2}i$.

Local review guidance: conjugate used to make a denominator real; exact modulus and argument values; Argand positions justified by geometry, not just plotted. This is authored classroom guidance.