9.5 Rank correlation and combinations of normals

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS Spearman's rank correlation coefficient

Rank correlation and combinations of normals

Objective

Compute and test Spearman's rank correlation, and combine independent Normal variables including differences and |differences| (objectives 1.1, 5.1–5.2).

1 Key knowledge and worked examples

Two toolkits. Spearman: $r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$; test against the table (H_0 $\rho = 0$). Combinations: $aX \pm bY \sim N(a\mu_x \pm b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2)$; for $P(|X-Y| > c)$, double the one tail.

Worked example (WST03 June 2025 Q2, Q8ab). (i) Judges' ranks with $\sum d^2 = 66$: $r_s = 1 - \frac{396}{990} = 0.6 < 0.7455$ (1% one-tailed) – insufficient evidence. (ii) $S \sim N(18, 0.03^2)$, $C \sim N(25, 0.03^2)$: $S_1 + S_2 + S_3 \sim N(54, 0.0027)$, so $P(\text{total} > 54.1) = 1 - \Phi(1.924) = 0.0274$; $C_1 - C_2 \sim N(0, 0.0018)$, so $P(|C_1 - C_2| > 0.02) = 2(1 - \Phi(0.4714)) = 0.638$.

2 Practice

2.1 Two rankings of 8 items differ by $d = 1, 0, 2, 1, 1, 0, 1, 0$: find r_s. [2]

2.2 Independent $X \sim N(10, 4)$ and $Y \sim N(6, 9)$: write down the distribution of $2X - Y$. [3]

2.3 For those variables, find $P(X - Y > 6)$. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WST03 June 2025 Q2.)* At a talent show two judges ranked 10 performances (Judge 1: A–J as 1, 3, 5, 7, 9, 10, 2, 6, 8, 4; Judge 2: 2, 1, 3, 7, 9, 4, 5, 8, 10, 6). (a) Calculate Spearman's rank correlation coefficient. (b) Stating your hypotheses, test at the 1% level whether there is evidence of a positive correlation between the judges' rankings. [7]

3.2 *(Edexcel IAL WST03 June 2025 Q8a–b, condensed.)* The weights of bags of sand are Normally distributed with mean 18 kg and standard deviation 0.03 kg, and bags of cement with mean 25 kg and standard deviation 0.03 kg (all independent). (a) Three bags of sand are selected at random: find the probability their total weight exceeds 54.1 kg. (b) Two bags of cement are selected: find the probability their weights differ by more than 0.02 kg. [9]

Solutions

2.1 $\sum d^2 = 1+0+4+1+1+0+1+0 = 8$: $r_s = 1 - \frac{48}{8(63)} = 1 - 0.0952 = 0.905$ (3 dp).

2.2 $2X - Y \sim N(2(10) - 6,\ 4(4) + 9) = N(14, 25)$.

2.3 $X - Y \sim N(4, 13)$: $z = \frac{6-4}{\sqrt{13}} = 0.555$, so $P = 1 - \Phi(0.555) = 1 - 0.7103 = 0.2897$.

3.1 (a) The rank differences (Judge 1 − Judge 2) are $-1, 2, 2, 0, 0, 6, -3, -2, -2, -2$, so $\sum d^2 = 1+4+4+0+0+36+9+4+4+4 = 66$ and $r_s = 1 - \frac{6(66)}{10(99)} = 1 - \frac{396}{990} = 0.6$. (b) $\mathrm{H}_0\colon \rho = 0$, $\mathrm{H}_1\colon \rho > 0$. The 1% one-tailed critical value for $n = 10$ is 0.7455. Since $0.6 < 0.7455$, do not reject H_0 – insufficient evidence of a positive correlation between the two judges' rankings.

3.2 (a) $S_1 + S_2 + S_3 \sim N(54, 3(0.0009)) = N(54, 0.0027)$, standard deviation $\sqrt{0.0027} = 0.05196$. $z = \frac{54.1-54}{0.05196} = 1.924$, so $P = 1 - \Phi(1.924) = 1 - 0.9726 = 0.0274$. (b) $C_1 - C_2 \sim N(0, 2(0.0009)) = N(0, 0.0018)$, standard deviation $\sqrt{0.0018} = 0.04243$. By symmetry, $P(|C_1 - C_2| > 0.02) = 2\left(1 - \Phi\left(\frac{0.02}{0.04243}\right)\right) = 2(1 - \Phi(0.4714)) = 2 \times 0.3192 = 0.638$.

Local review guidance: $\sum d^2$ shown term by term; critical value from the Spearman table with the correct tail; combined variances with squared coefficients; the doubling for an absolute difference. This is authored classroom guidance.