

## 8.6 Normal approximations and thresholds

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 22 marks

**KEY WORDS** continuity correction • normal approximation

### Normal approximations and thresholds

#### Objective

Approximate binomial (and Poisson) distributions with the Normal distribution, apply the continuity correction, solve threshold problems for  $r$  and  $t$ , and combine with testing (objectives 1.3, 3.2, 4.6).

#### 1 Key knowledge and worked examples

**Approximation.** For large  $n$ :  $B(n, p) \approx N(np, np(1 - p))$  (Poisson when instead  $p$  is small). "At least  $t$ " uses  $t - 0.5$ ; "at most  $r$ " uses  $r + 0.5$ . Threshold searches: accumulate exact binomial probabilities for small  $n$ ; standardise with the correction for large  $n$  and invert  $z = \frac{x - 0.5 - \mu}{\sigma}$ .

**Worked example – thresholds both ways (WST02 June 2025 Q7c–d).**  $X \sim B(40, 0.3)$ : largest  $r$  with  $P(X < r) < 0.05$  (fewer than  $r$ ):  $P(X \leq 6) = 0.0238 < 0.05$  but  $P(X \leq 7) = 0.0553 > 0.05$ , so  $r = 7$ . For 200 customers:  $B(200, 0.3) \approx N(60, 42)$ ;  $P(X \geq t) = 0.9474$  means  $\frac{t - 0.5 - 60}{\sqrt{42}} = -1.62$ , so  $t = 0.5 + 60 - 1.62\sqrt{42} = 50.001$ :  $t = 50$ .

#### 2 Practice

**2.1**  $X \sim B(120, 0.4)$ . State the approximating Normal distribution.

[2]

**2.2** Using that approximation, find  $P(X > 55)$ , with continuity correction. [3]

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**2.3** A Poisson variable has mean 100. State its Normal approximation and estimate  $P(X < 90)$ . [3]

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### 3 Exam-style questions

**3.1** (*Edexcel IAL WST02 June 2025 Q7, condensed.*) 30% of a random sample of 40 customers buying glasses also buy insurance;  $X$  is the number buying insurance. (a) State the distribution of  $X$  and one necessary assumption. (b) Find the largest  $r$  for which  $P(X < r) < 0.05$  (fewer than  $r$ ). (c) For a second sample of 200 customers, using a suitable Normal approximation, find the value of  $t$  such that  $P(\text{at least } t \text{ of the 200 buy insurance}) = 0.9474$  (4 dp), showing all working. [9]

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**3.2** (*Original.*) A telephone line receives calls at an average rate of 4 per minute. Using a Normal approximation, estimate the probability that fewer than 200 calls arrive in one hour. [5]

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## Solutions

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**2.1**  $N(48, 28.8)$  (mean  $120 \times 0.4$ ; variance  $120 \times 0.4 \times 0.6 = 28.8$ ).

**2.2**  $P(X > 55) = P(Z > \frac{55.5-48}{\sqrt{28.8}}) = P(Z > 1.398) = 1 - 0.9192 = 0.081$ .

**2.3**  $N(100, 100)$ ;  $P(X < 90) = P(Z < \frac{89.5-100}{10}) = P(Z < -1.05) = 0.1469$  (0.147).

**3.1** (a)  $X \sim B(40, 0.3)$ ; customers buy insurance independently with a constant probability 0.3. (b)  $P(X < r) = P(X \leq r - 1)$ :  $P(X \leq 6) = 0.0238 < 0.05$  but  $P(X \leq 7) = 0.0553 > 0.05$ , so the largest such  $r$  is 7 (P(fewer than 7) is the last one under 0.05) (c)  $B(200, 0.3) \approx N(60, 42)$  with  $\sigma = \sqrt{42}$ . "At least  $t$ "  $\rightarrow$  boundary  $t - 0.5$ :  $P\left(Z > \frac{t-0.5-60}{\sqrt{42}}\right) = 0.9474$  means  $\frac{t-0.5-60}{\sqrt{42}} = -1.62$ , so  $t = 0.5 + 60 - 1.62(6.481) = 50.001$ :  $t = 50$ .

**3.2** Calls per hour:  $Po(240) \approx N(240, 240)$  with  $\sigma = \sqrt{240} = 15.49$ . Fewer than 200  $\rightarrow$  boundary 199.5:  $P\left(Z < \frac{199.5-240}{15.49}\right) = P(Z < -2.615)$ . From tables,  $\Phi(-2.62) = 0.0044$ : the estimate is awrt 0.0044.

Local review guidance: the correct  $\pm 0.5$  boundary for the phrasing ("at least", "fewer than"); z-values to 2 dp and inverted cleanly to the threshold;  $r$ -searches closed by the two straddling probabilities, both quoted. This is authored classroom guidance.