8.5 Hypothesis tests

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS null hypothesis • critical region

Hypothesis tests

Objective

Run one- and two-tailed tests for a Poisson rate or a binomial proportion: hypotheses, critical region by accumulation, actual significance level, in-context conclusion (objectives 4.3–4.6).

1 Key knowledge and worked examples

The routine. H_0: parameter equals the claimed value; H_1: $\neq$, $<$ or $>$ as asked. Accumulate tail probabilities until crossing the level α; the critical region is the complement of the largest non-crossing set; the actual significance level is the total tail probability of the region. Conclude in context with "insufficient evidence" phrasing when the observation is outside the region.

Worked example – a two-tailed Poisson test (WST02 June 2025 Q2). $X \sim \text{Po}(10)$ per 30 minutes, $H_0\colon \lambda = 10$, $H_1\colon \lambda \neq 10$, 3% level: $P(X \leq 3) = 0.0103$, $P(X \leq 4) = 0.0293$; $P(X \geq 17) = 0.0270$, $P(X \geq 18) = 0.0143$ – critical region $X \leq 3$ or $X \geq 18$, actual level 2.46%. Observed 16: outside $\rightarrow$ do not reject H_0.

2 Practice

2.1 A test at the 5% level ends with "reject H_0". State, in one sentence each, what rejecting and not rejecting mean in terms of evidence. [2]

2.2 $X \sim B(30, 0.5)$ under H_0; the test is $H_1\colon p > 0.5$ at 5%. Given $P(X \geq 20) = 0.0494$ and $P(X \geq 19) = 0.1002$, state the critical region. [2]

2.3 For that test, state the actual significance level. [1]

3 Exam-style questions

3.1 *(Edexcel IAL WST02 June 2025 Q2.)* Customers enter a shop between 7 pm and 11 pm on Fridays at a mean rate of 20 per hour. After shop alterations, the manager monitors ONE randomly chosen 30-minute interval on the following Friday; 16 customers enter. (a) State the distribution to model the number of customers entering in a 30-minute interval. (b) State one necessary assumption. (c) Write down suitable null and alternative hypotheses. (d) Using a 3% level of significance, find the critical region. (e) Find the actual significance level. (f) Comment on the manager's observation. [10]

3.2 *(Edexcel IAL WST02 June 2025 Q7e, condensed.)* Before a special offer, 30% of a shop's customers buying glasses also took insurance. After the offer (first 3 months free), a random sample of 25 customers is taken and 11 buy insurance. Test, at the 10% level of significance, whether the proportion of customers buying insurance has increased. State your hypotheses clearly. [5]

Solutions

2.1 Rejecting H_0: the sample provides sufficient evidence (at that level) that the parameter has changed as H_1 claims. Not rejecting: insufficient evidence – not proof that H_0 is true.

2.2 $P(X \geq 20) = 0.0494 < 0.05$ and $P(X \geq 19) = 0.1002 > 0.05$, so the critical region is $X \geq 20$.

2.3 $0.0494 = 4.94\%$.

3.1 (a) $X \sim \text{Po}(10)$ (20 per hour $\to$ 10 per half hour). (b) Customers enter singly (independently) at a constant average rate. (c) $H_0\colon \lambda = 10$, $H_1\colon \lambda \neq 10$ (equivalently 20 vs $\neq 20$ per hour). (d) Lower tail: $P(X \leq 3) = 0.0103$, $P(X \leq 4) = 0.0293 > 0.015$ – stop at 3. Upper tail: $P(X \geq 17) = 0.0270 > 0.015$, $P(X \geq 18) = 0.0143$ – stop at 18. Critical region: $X \leq 3$ or $X \geq 18$. (e) $0.0103 + 0.0143 = 0.0246 = 2.46\%$. (f) 16 lies outside the critical region: do not reject H_0 – insufficient evidence that the rate of customers entering has changed.

3.2 $H_0\colon p = 0.3$, $H_1\colon p > 0.3$; under H_0, $X \sim B(25, 0.3)$. $P(X \geq 11) = 1 - P(X \leq 10) = 1 - 0.9022 = 0.0978 < 0.10$, so 11 lies in the critical region ($X \geq 11$): reject H_0 – there is evidence, at the 10% level, that the proportion of customers buying insurance has increased.

Local review guidance: hypotheses with the parameter named in symbols and words; tail accumulation on the correct side(s); actual level as the sum of exactly the tails used; conclusions in context, "insufficient evidence" when outside. This is authored classroom guidance.